

Mathematical Model Development for Design Improvement of a Gas-Fired Pyrolysis Reactor

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Abstract: Conversion of wastes into valuable resources may not be effective with defective equipment. The aim of the study was to develop a mathematical model using data generated from the laboratory study of thermal efficiencies and air pollution impacts of locally fabricated liquefied petroleum gas (LPG) burners. This was in order to obtain optimum number of burner holes and air-to-fuel (LPG) ratio for design improvement of air-fuel intake port of a gas-fired pyrolysis reactor. The data were modeled for the effects of burner hole type, fuel flow rate and air-fuel ratio; on the thermal efficiencies and emission characteristics of the LPG gas burners. Regression model for thermal efficiencies gave a good fitness to experimental data and is significant for predicting thermal efficiency response variable with high correlation coefficient of 99.97%. Predicted data for thermal efficiency gave highest value of 69% when LPG flowrate and burner hole type were at 1.0 litre/min and 144 respectively. Analysis of characteristic emissions from the gas burners including CO, NO_x and TSP emissions showed that environmental effect of combustion using the gas burners is minimal when operating at highest gas flowrate and burner hole type. From the results of the data modeling, optimum thermal efficiency, air-to-fuel ratio and lowest emissions were predicted when burner hole type and gas flowrate were optimal at 144 and 1.0 litre/min.

Keywords: Mathematical modeling, pyrolysis, gas burners, air-fuel ratio, emissions, thermal efficiency

I. Introduction

The ineffective management of disposal of non-biodegradable polymeric wastes, including rubber tyres, plastic bottles, diapers, nylon wastes, among others; is a major challenge in increased rate of consumption of the polymeric materials in recent times. Disposal of the non-biodegradable polymeric wastes through land filling, open burning or ocean dumping is associated with damaging environmental and health implications. For example, burning of tyres outdoor, leads to the release of large amount of dangerous, toxic and carcinogenic inorganic substances into the atmosphere while tyres buried underground decompose under natural conditions for more than 100 years [1]. The contact of these decomposing tyres with rainwater and groundwater leads to the formation of organic toxins and carcinogenic chemical compounds. To minimize the negative effects of improper disposal of non-biodegradable polymeric wastes, different environmentally-friendly techniques, including pyrolysis, have been developed to manage the

disposal of the polymeric wastes. Pyrolysis is the thermal or catalytic decomposition of materials at elevated temperatures in an inert atmosphere or in the absence of oxygen. It is a technique that is being used as an environmental friendly tool for waste valorization [2-6]. Pyrolysis is an effective process for the control of environmental pollution caused by solid waste materials especially non-biodegradable polymeric wastes. Besides the environmental benefits, valuable products such as activated carbon, diesel oil, fuel gases, bitumen, among others can be obtained from the pyrolysis technique [6-11].

Meanwhile, developing and designing a pyrolysis process to meet its specification and requirement entails provision and analyses of mathematical models which will describe its kinetics, mechanism and optimization [12]. Pyrolysis models are gaining importance not only because it is studied as an independent process but because it is an initial step in gasification or combustion process [12]. Several researchers have carried out different studies on

mathematical modeling of pyrolysis process using different computer software packages. For example, Zheleva et al.[1], studied the mathematical modeling of heat transfer during pyrolysis process used for the treatment of end-of-life tyres. The researchers used MATLAB to develop an algorithm for solving 2D models created for temperature of non-stationary heat transfer of the pyrolysis process. Results for the temperature characteristic periods of operation of the pyrolysis process obtained from the model showed quality compliance with the actual pyrolysis process. Also, FORTRAN programming language was used by Srivastava et al. [12], to develop subroutines for results during initial and final stages of pyrolysis when they studied the development of mathematical model for the prediction of concentration in the pyrolysis of biomass materials. Srivastava and his team of researchers concluded that pyrolysis is much faster when model differential equations are independent of initial biomass concentration as oppose to when the equations are dependent on initial concentration of biomass.

The present research used statistical analysis software called MINITAB (version 17.0) to model the combustion process in the furnace of a gas-fired pyrolysis reactor in order to determine the optimum burner hole and air-to-fuel ratio for the improvement of the design of air-fuel intake port of the gas-fired pyrolysis reactor. The model was based on the data generated from the laboratory study of thermal efficiencies and air pollution impacts of locally fabricated liquefied petroleum gas (LPG) burners [13]. The choice of this software tool is due to its accuracy of presenting statistical result and ease of use compared to other statistical packages.

II. Materials and Methods

Three heating stoves were designed and fabricated using galvanized iron sheet. The stoves were designed to burn liquefied petroleum gas using regulated amount of air for combustion to produce luminous blue flame. The number of burner holes on

stoves 1, 2 and 3 were 48, 96 and 144 respectively. The experimental procedure was conducted in two phases. During the first phase, the effects of variation in number of burner holes and gas flow rates on thermal efficiencies of the stoves were determined. While in the second phase, investigation of the effects of variation in number of burner holes and gas flow rates on emission characteristics of the three stoves were carried out. Data generated from experiment (Table 1) was then adapted and modeled using the Fit Regression feature of Statistical Analysis Button of MINITAB v17.0.

III. Results and Discussion

Results of original experiment and that generated from statistical analysis of experimental data using MINITAB v17.0 are shown in Tables 1 to 9. Table 1 shows variations of the independent variable, LPG (gas/fuel) flowrate with various dependent variables in relationship to heating medium number of burner holes or burner hole type (BT). Data from Table 1 showed that energy efficiency and air-to-fuel ratio values for burner hole type, BT = 144 was higher than BT = 96 which in turn was higher than BT = 48 for the different LPG flow rates. The heating medium burner hole type showed similar trends with LPG flowrates in relation to other dependent variables.

3.1. Model development

The model development was in two phases; the first phase focused on the effects of variation in number of burner hole types and gas flowrates on thermal efficiencies while the second phase focused on the effects of variation in number of burner hole types (BT) and gas flowrates (Q) on emission characteristics of heating medium or gas burners.

3.1.1. Model development for first phase (Phase 1): Effects of variation in number of burner hole types and gas flowrates on thermal efficiencies.

The model for thermal efficiency (E (%)) of the gas burners with different burner holes, was developed with the Fit Regression feature of MINITAB v17.0 statistical software. This model is the regression line equation given in Equation 1:

$$E(\%) = 48.81 - 7.4Q + 0.2315 + 7.62Q^2 - 0.000521BT^2 - 0.0188QBT \quad (1)$$

With the regression model above, values for response variable, thermal efficiency (E (%)) were predicted using LPG (gas) flowrates and heating medium burner hole type (BT) as predictor variables. Results are shown in Table 2.

Table 1: Data generated from the laboratory study of thermal efficiencies and air pollution impacts of locally fabricated liquefied petroleum gas burners

<i>Independent variable (Fuel(LPG) flow rate, litres/min)</i>			0.80	0.85	0.90	0.95	1.00
S/N	Dependent variable	Burner type					
1	Velocity of gas-air mixture (m/s)	Stove 1(n=48)	0.863	0.923	0.960	1.210	1.270
		Stove 2(n=96)	0.458	0.494	0.574	0.607	0.690
		Stove 3(n=144)	0.361	0.389	0.427	0.449	0.480
2	Rate of heating (kJ/s)	Stove 1(n=48)	1.25	1.33	1.41	1.49	1.57
		Stove 2(n=96)	1.25	1.33	1.41	1.49	1.57
		Stove 3(n=144)	1.25	1.33	1.41	1.49	1.57
3	Volume of fuel (m ³)	Stove 1(n=48)	40,160 x 10 ⁻⁶	39,780 x 10 ⁻⁶	39,330 x 10 ⁻⁶	38,665 x 10 ⁻⁶	38,000 x 10 ⁻⁶
		Stove 2(n=96)	30,336 x 10 ⁻⁶	29,750 x 10 ⁻⁶	28,980 x 10 ⁻⁶	28,690 x 10 ⁻⁶	28,000 x 10 ⁻⁶
		Stove 3(n=144)	24,000 x 10 ⁻⁶	23,588 x 10 ⁻⁶	23,040 x 10 ⁻⁶	22,468 x 10 ⁻⁶	22,000 x 10 ⁻⁶
4	Volume of gas-air mixture (m ³)	Stove 1(n=48)	391,560 x 10 ⁻⁶	390,640 x 10 ⁻⁶	377,961 x 10 ⁻⁶	445,808 x 10 ⁻⁶	437,000 x 10 ⁻⁶
		Stove 2(n=96)	314,888 x 10 ⁻⁶	313,268 x 10 ⁻⁶	335,009 x 10 ⁻⁶	330,796 x 10 ⁻⁶	350,000 x 10 ⁻⁶
		Stove 3(n=144)	294,000 x 10 ⁻⁶	292,721 x 10 ⁻⁶	296,986 x 10 ⁻⁶	288,483 x 10 ⁻⁶	286,000 x 10 ⁻⁶
5	Volumetric flow rate (m ³ /s)	Stove 1(n=48)	1.30 x 10 ⁻⁴	1.39 x 10 ⁻⁴	1.44 x 10 ⁻⁴	1.83 x 10 ⁻⁴	1.92 x 10 ⁻⁴
		Stove 2(n=96)	1.38 x 10 ⁻⁴	1.49 x 10 ⁻⁴	1.73 x 10 ⁻⁴	1.83 x 10 ⁻⁴	2.08 x 10 ⁻⁴
		Stove 3(n=144)	1.63 x 10 ⁻⁴	1.76 x 10 ⁻⁴	1.93 x 10 ⁻⁴	2.03 x 10 ⁻⁴	2.17 x 10 ⁻⁴
6	Boiling time (min)	Stove 1(n=48)	50.20	46.80	43.70	40.70	38.00
		Stove 2(n=96)	37.92	35.00	32.20	30.20	28.00
		Stove 3(n=144)	30.00	27.75	25.60	23.65	22.00
7	Air flow rate (litres/min)	Stove 1(n=48)	7.00	7.50	7.75	10.00	10.50
		Stove 2(n=96)	7.50	8.10	9.50	10.0	11.50
		Stove 3(n=144)	9.00	9.70	10.7	11.25	12.00
8	Air to fuel ratio	Stove 1(n=48)	8.75	8.82	8.61	10.53	10.50
		Stove 2(n=96)	9.38	9.53	10.56	10.53	11.50
		Stove 3(n=144)	11.25	11.41	11.89	11.84	12.00
9	Thermal Efficiency (%)	Stove 1(n=48)	56.9	57.1	57.5	57.9	58.0
		Stove 2(n=96)	63.9	64.0	64.1	64.3	64.6
		Stove 3(n=144)	68.1	68.3	68.4	68.6	69.0
10	Energy intensity (kJ/g of water)	Stove 1(n=48)	1.89	1.87	1.85	1.82	1.79

		Stove 2(n=96)	1.43	1.40	1.36	1.35	1.32
		Stove 3(n=144)	1.13	1.11	1.08	1.06	1.03
11	Energy consumed (kJ)	Stove 1(n=48)	3,775	3,739	3,697	3,635	3,572
		Stove 2(n=96)	2,852	2,797	2,724	2,697	2,632
		Stove 3(n=144)	2,256	2,217	2,166	2,112	2,068
12	Carbon monoxide (CO) emission (mg/m ³)	Stove 1(n=48)	34.43	42.17	48.50	351.32	204.69
		Stove 2(n=96)	145.20	44.47	39.60	43.85	46.41
		Stove 3(n=144)	135.82	161.14	269.45	271.03	212.00
13	NOx emission (mg/m ³)	Stove 1(n=48)	19.09	23.60	15.83	25.68	20.52
		Stove 2(n=96)	16.34	19.77	20.41	20.37	17.57
		Stove 3(n=144)	19.71	24.47	31.96	18.83	18.83
14	Total Suspended Particle (TSP) emission (µg/m ³)	Stove 1(n=48)	69,091	50,000	34,286	85,714	55,385
		Stove 2(n=96)	40,000	28,800	57,143	25,455	41,739
		Stove 3(n=144)	40,000	25,000	47,293	53,333	28,144

TABLE 2. Model prediction data for thermal efficiency, E (%).

Q (gas flow rates)	BT (Burner hole type)	E (%)
0.80	48	56.9
0.80	96	63.9
0.80	144	68.1
0.85	48	57.1
0.85	96	64.0
0.85	144	68.3
0.90	48	57.5
0.90	96	64.1
0.90	144	68.4
0.95	48	57.9
0.95	96	64.3
0.95	144	68.6
1.00	48	58.0
1.00	96	64.6
1.00	144	69.0

From Table 2, it could easily be seen that the burner hole type (BT) of 144 burner holes gave higher thermal efficiency values using the model than BT of 96 burner holes, which in turn gave higher values than BT of 48 burner holes for all LPG (gas) flowrates.

ANOVA analysis was carried out on experimental data to determine effect of predictor variables, including LPG flowrates (Q) and gas burner hole type (BT) on response variable, thermal efficiency (E (%)). ANOVA table for E (%) is presented in Table 3. Table 3 showed that probability values (p-value) obtained from E (%) regression model are greater than 0.05 which is the benchmark significant level value (α -value). This means that LPG flowrates (Q) and heating device burner hole types (BT) have significant effect on the thermal efficiency using the regression model. Also, the model summary in Table 3 showed a low standard error (S) of 0.109; this implies that the regression model effectively described the response variable- thermal efficiency. The high percentage (99.97%) of correlation coefficient (R-sq) from TABLE 3 showed that the developed regression model fits experimental data in TABLE 1. In the same vein, predicted R-sq and adjusted R-sq with values of 99.88% and 99.95% respectively, indicate that the regression model will very well predict response variable (thermal efficiency) for new experimental data and it (model) can explain 99.95% variance in the response variable. Relationship between the response variable E (%) and predictor variables Q and BT can further be shown on the contour (Figure 1) and surface (Figure 2) plots.

Figure 1, shows the contour plot representation for Thermal efficiency (E (%)) prediction. This showed a 3-dimensional plot of the relationship between predictor variables (flow rate of the gas and burner hole

Table 3 : ANOVA table for E (%) model prediction.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	308.897	61.7794	5199.87	0.000
Q	1	0.004	0.0044	0.37	0.559
BT	1	5.850	5.8499	492.37	0.000
Q*Q	1	0.015	0.0152	1.28	0.287
BT*BT	1	4.800	4.8000	404.01	0.000
Q*BT	1	0.041	0.0405	3.41	0.098
Error	9	0.107	0.0119		
Total	14	309.004			
Model Summary:					
S	R-sq	R-sq(adj)	R-sq(pred)		
0.109000	99.97%	99.95%	99.88%		
Coefficients:					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	48.81	5.50	8.87	0.000	
Q	-7.4	12.2	-0.61	0.559	932.71
BT	0.2315	0.0104	22.19	0.000	211.00
Q*Q	7.62	6.73	1.13	0.287	926.71
BT*BT	-0.000521	0.000026	-20.10	0.000	49.00
Q*BT	-0.0188	0.0102	-1.85	0.098	169.00

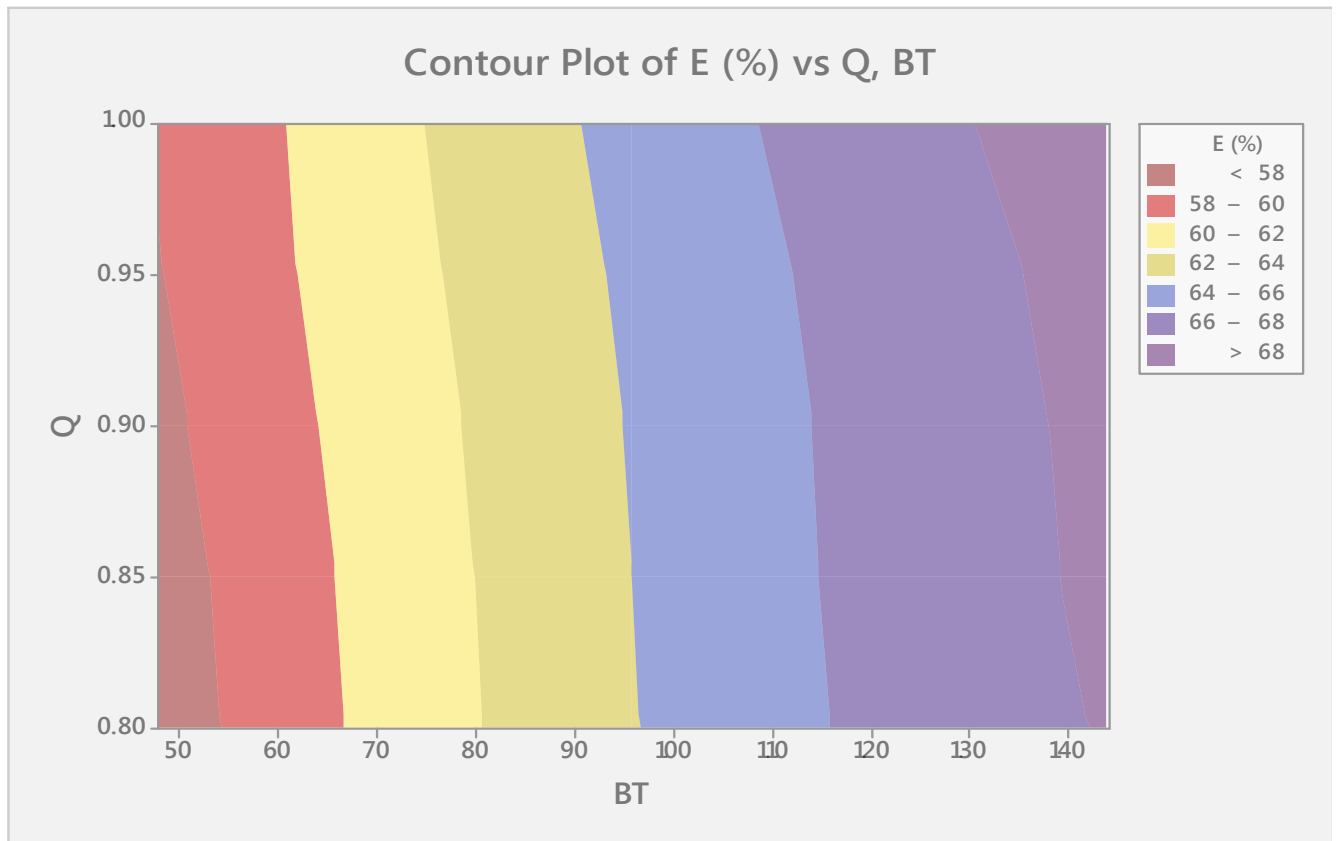


Figure 1: Contour plot for thermal efficiency prediction.

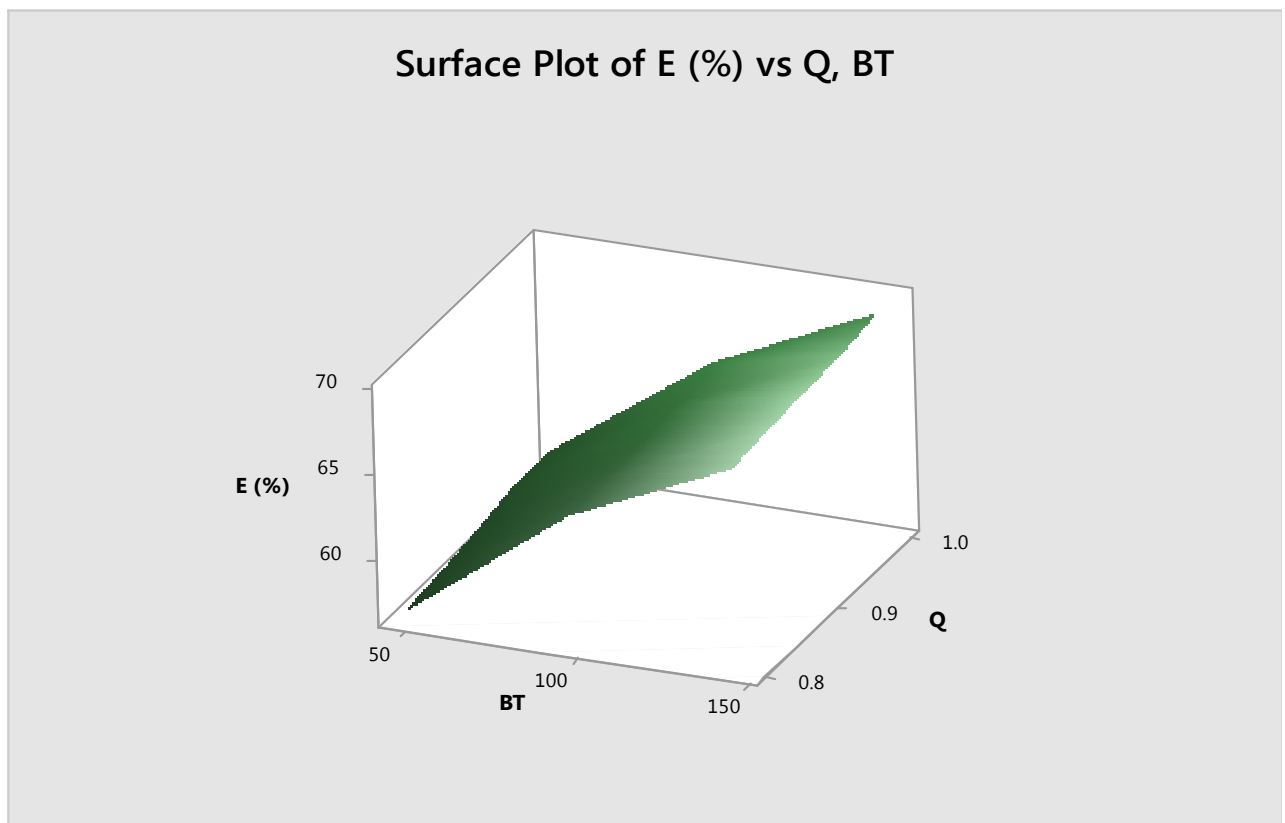


Figure 2: Surface plot for thermal efficiency prediction.

type) and response variable (thermal efficiency) on a 2-dimensional plane with a constant Z. The plot is depicted such that burner hole type (BT) and gas flowrate (Q) values are plotted on the x- and y-planes and response values (E (%)) are represented by the contours, constant Z-lines. From the plot, E (%) is highest (> 68%) with purple colour contour line when Q was 1.00 litre/min and BT was >140; and lowest (< 58%) with brown contour line when Q was 0.80 litre/min and BT was < 50. Also, as an accompanying plot to the contour plot, Figure 2 shows surface plot for the relationship between E (%), Q and BT. It is a 3-dimensional graphical representation, with predictor variables on x- and y-axes, and the response variable on z-axis. It is generated from the experimental results for prediction of the response variable from flow rate of gas and burner hole type. Like the contour plot, the surface plot showed that E (%) was highest when Q and BT were high.

3.1.2. Model development for second phase (Phase 2): Effects of variation in number of burner hole types and gas flowrates on emission characteristics of gas burners.

The effect of predictor variables, BT and Q on response variable, emission characteristics of the gas burners was modelled using regression analysis with MINITAB v17. Regression model was developed for each emission from the gas burners and these include carbon monoxide (CO), nitrogen oxides (NO_x) and total suspended particles (TSP).

Regression and ANOVA analyses for Carbon monoxide (CO) emission:

The regression model developed with MINITAB v17 for CO emission is given in Equation 2 while ANOVA analysis for CO emission is given in Table 4.

$$CO = -1255 + 2495Q - 1.06BT - 690Q^2 + 0.0474BT^2 - 8.08QBT \quad (2)$$

From Table 4, it can be seen that p-values are greater than 0.05 (significance level value); this means that LPG flowrates and burner hole types have significant effect on CO emission from the pyrolysis process. The correlation coefficient (R-sq) has a value of 50.35% indicating that the CO model averagely fits experimental data. The standard error (S) and predicted R-sq of 92.4112 and 0.00% respectively, indicate that the model do not effectively predict response variable, CO. Figures 3 and 4 are the contour plot and corresponding surface plot for CO emission prediction. Figure 3 showed that CO emission was lowest (< 50 mg/m³) with a brown contour line, at the start of experiment when LPG flowrate (Q) was lowest (0.8 litre/min) and burner hole type (BT) was < 50; but increased to highest emissions (> 350 mg/m³) with a purple contour line during experiment, when Q was 0.9 < Q ≤ 0.95 and BT was 50 < BT < 100. The CO emission, as shown by contour plot, gradually reduced (200 < emission < 250 mg/m³) with green contour line, when Q was highest at 1.0 litre/min and BT was > 140. The corresponding 3-dimensional surface plot for CO prediction, Figure 4, showed that CO emission was highest at > 300 mg/m³ when Q was < 1.0 litre/min and burner hole type was < 100.

Table 4: ANOVA table for CO emission model prediction.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	77931	15586.2	1.83	0.204
Q	1	500	500.4	0.06	0.814
BT	1	123	122.7	0.01	0.907
Q*Q	1	125	125.1	0.01	0.906
BT*BT	1	39748	39748.1	4.65	0.059
Q*BT	1	7524	7524.1	0.88	0.372
Error:	9	76858	8539.8		
Total:	14	154789			
Model Summary:					
S	R-sq	R-sq(adj)	R-sq(pred)		
92.4112	50.35%	22.76%	0.00%		
Coefficients:					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-1255	4665	-0.27	0.794	
Q	2495	10305	0.24	0.814	932.71
BT	-1.06	8.84	-0.12	0.907	211.00
Q*Q	-690	5704	-0.12	0.906	926.71
BT*BT	0.0474	0.0220	2.16	0.059	49.00
Q*BT	-8.08	8.61	-0.94	0.372	169.00

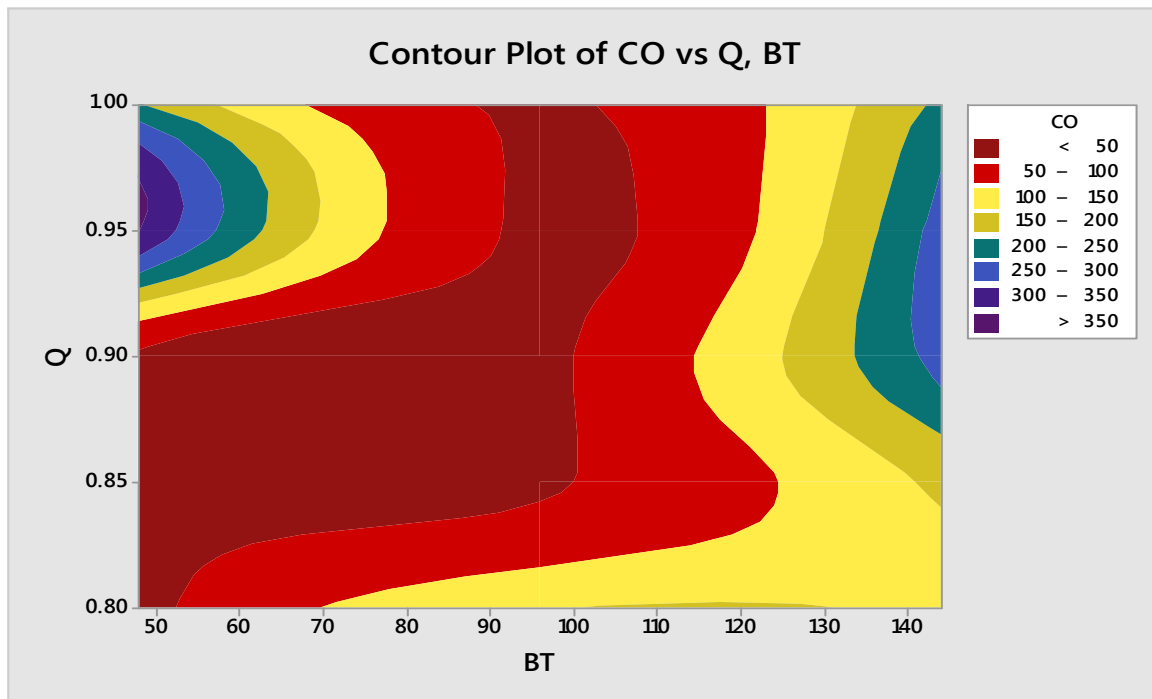


Figure 3: Contour plot for CO emission prediction

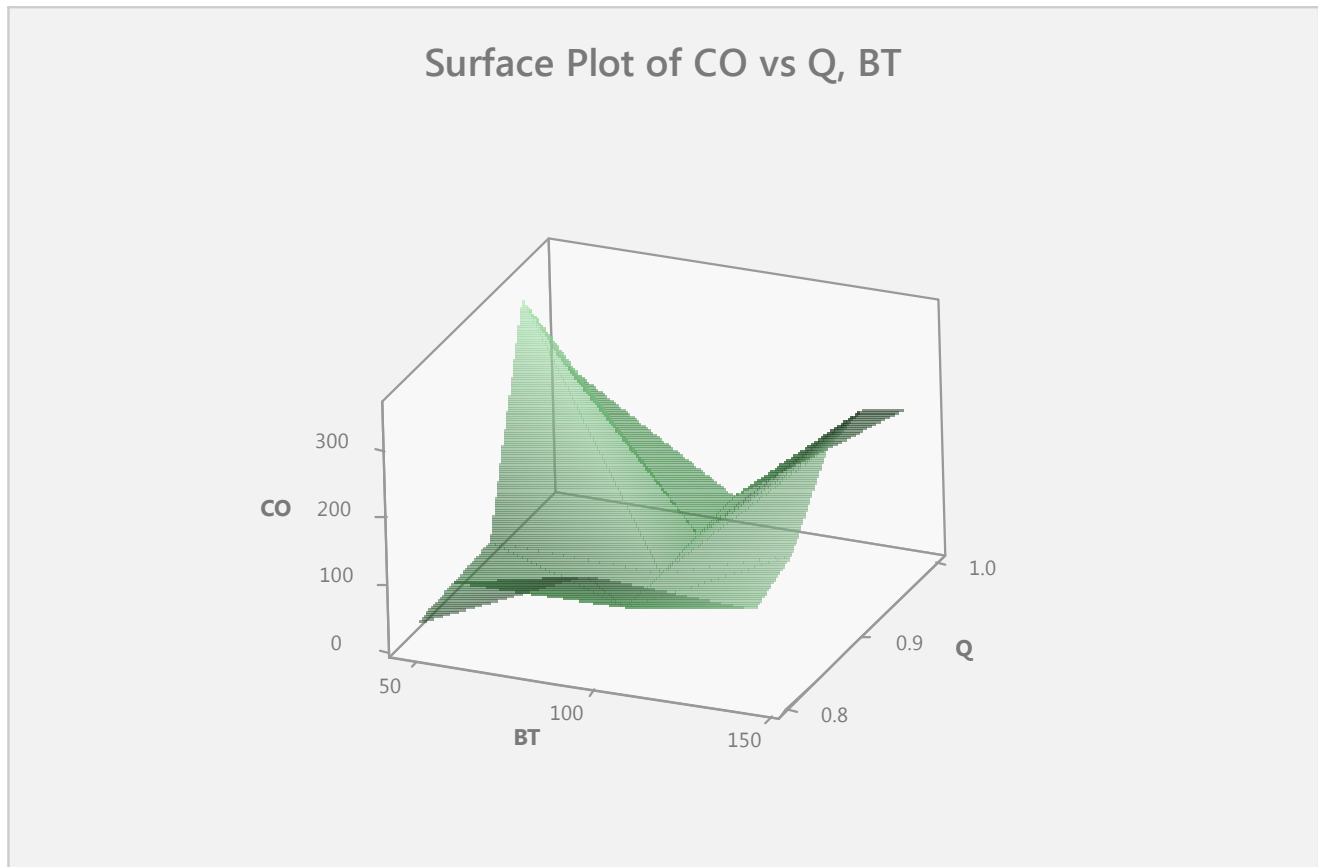


Figure 4: Surface plot for CO emission prediction

Regression and ANOVA analyses for oxides of Nitrogen (NO_x) emission:

The NO_x emission regression model is given in Equation 3 while the ANOVA result for NO_x emission is given in Table 5.

$$NO_x = -339 + 797Q + 0.004BT - 429Q^2 + 0.001285BT^2 - 0.257QBT \quad (3)$$

P-values from Table 5 are greater than 0.05 (significance level) which indicate that LPG flowrates and burner hole types have significant effect on NO_x emission. The fit of experimental data to NO_x regression model as shown by R-sq, is 39.20%. Low standard error value of 4.00958 showed that the model is a fair description of response variable NO_x; but does not predict NO_x since predicted R-sq has value of 0.00%. Contour and surface plots for NO_x emission are shown by Figures 5 and 6 respectively. In the contour plot of Figure 5 for NO_x emission, emission was lowest (< 18 mg/m³) with a brown contour line, at beginning of experiment when Q was 0.8 litre/min and BT was < 50; but increased to highest emissions (> 30 mg/m³) with purple contour line during experiment, when Q was 0.9 and BT was 50 < BT < 100.

Table 5: ANOVA table for NOx emission model prediction

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	93.290	18.6580	1.16	0.398
Q	1	51.014	51.0145	3.17	0.109
BT	1	0.001	0.0014	0.00	0.993
Q*Q	1	48.214	48.2143	3.00	0.117
BT*BT	1	29.205	29.2053	1.82	0.211
Q*BT	1	7.614	7.6138	0.47	0.509
Error	9	144.691	16.0767		
Total	14	237.981			
Model Summary:					
S	R-sq	R-sq(adj)	R-sq(pred)		
4.00958	39.20%	5.42%	0.00%		
Coefficients:					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	-339	202	-1.67	0.129	
Q	797	447	1.78	0.109	932.71
BT	0.004	0.384	0.01	0.993	211.00
Q*Q	-429	247	-1.73	0.117	926.71
BT*BT	0.001285	0.000953	1.35	0.211	49.00
Q*BT	-0.257	0.374	-0.69	0.509	169.00

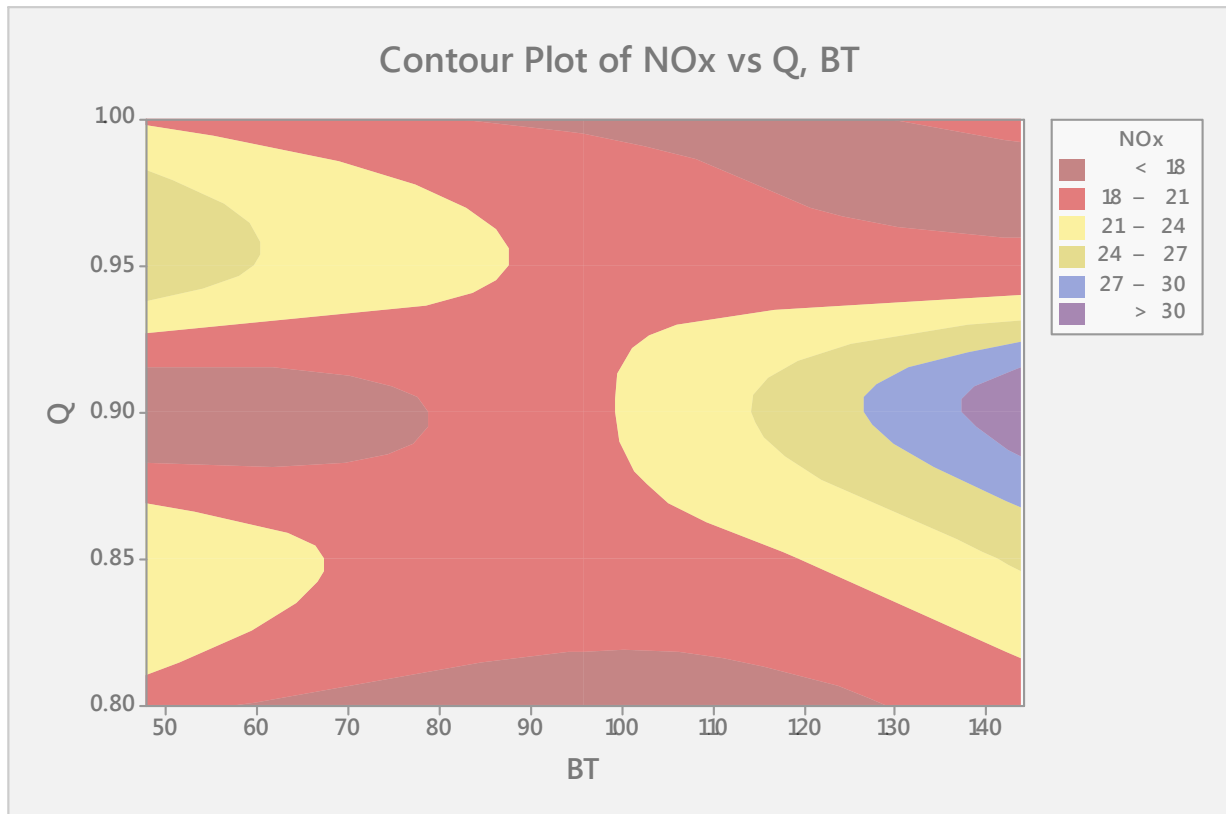


Figure 5: Contour plot for NOx emission prediction

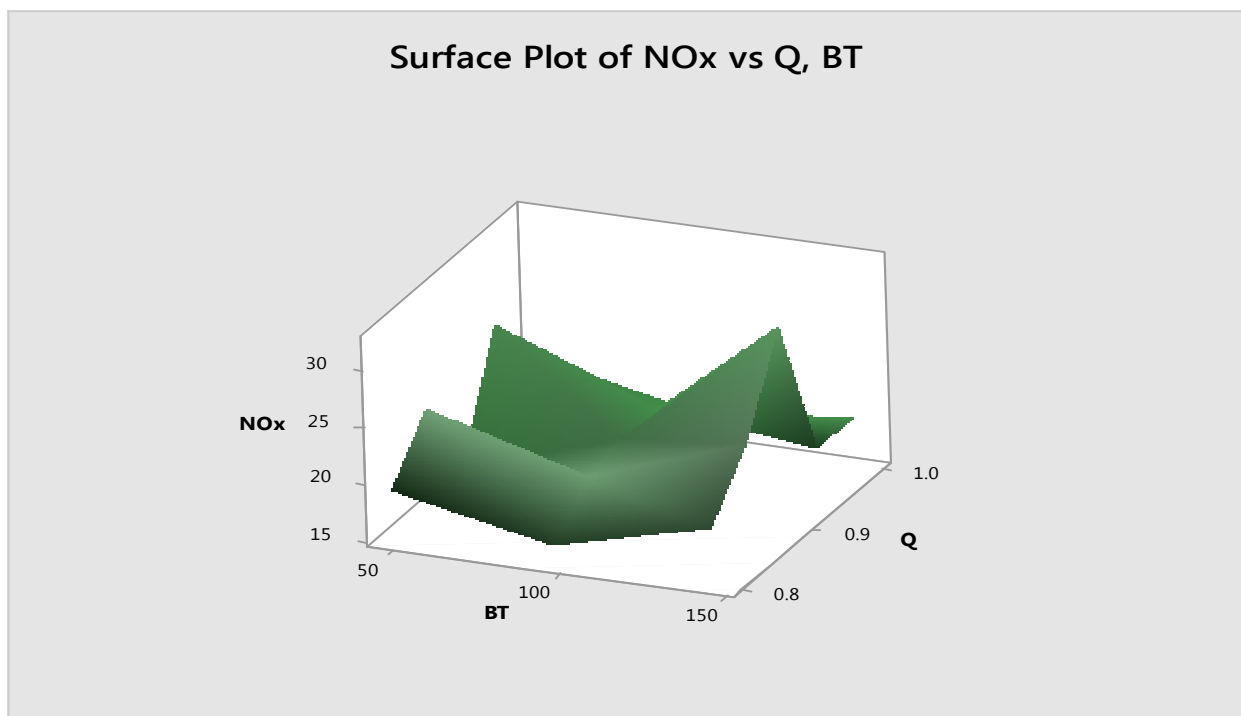


Figure 6: Surface plot for NOx emission prediction

NOx emission gradually reduced ($18 < \text{emission} < 21 \text{ mg/m}^3$) with red contour line, when Q was 1.0 litre/min and BT was > 140 . The surface plot, figure 6, showed that NOx emission was highest at $> 30 \text{ mg/m}^3$ when Q was

<0.9litre/min and burner hole type was between 50 and 100.

Regression and ANOVA analyses for Total Suspended Particle emission (TSP):

TSP regression model for Total Suspended Particle emission (TSP) is given in Equation 4 while the ANOVA analysis for TSP is presented in Table 6.

$$TSP = 114017 - 48064Q - 1004BT + 35924Q^2 + 4.50BT^2 - 77QBT \quad (4)$$

Table 6 showed that all p-values are greater than 0.05 (significance level value) which means that LPG flowrates and burner hole types have significant effect on TSP emission from the pyrolysis process. The TSP model fits experimental data at 33.46% R-sq value. Also Table 6.0 showed that standard error (S) and predicted R-square 17455.7 and 0.00% respectively, indicating that the model do not effectively predict response variable, TSP. Figures 7 and 8 are the contour plot and corresponding surface plot for TSP emission prediction. Figure 7 showed that suspended particle emission was between 60,000 and 70,000 mg/m³ with blue contour line at beginning of experiment when Q was 0.8 litre/min and BT was < 50; but increased to highest emissions (>80,000 mg/m³) with purple contour line during experiment, when Q was 0.95 and BT was 50<BT<100. Suspended particle emission reduced below 30,000 mg/m³ with brown contour line, when Q was 1.0 litre/min and BT was > 140. The surface plot, Figure 8, showed that TSP emission was highest at >80,000mg/m³ when Q was < 0.9 litre/min and burner hole type was between 50 and 100.

The summary of data obtained from the regression models for the emission characteristics of the pyrolysis process is given in Table 7. It is evident from data in Table 7 that values for each characteristic emission obtained from the models fluctuated with high values 145.20 mg/m³, 19.09 mg/m³ and 69,091mg/m³ obtained for CO, NO_x and TSP respectively, at start of experiment when Q was 0.8 and BT was 96 and increased to 351.82 mg/m³, 25.68 mg/m³ and 85,714 mg/m³ during the experiment when Q was 0.95 and BT was 48 but decreased at the end of experiment to 212.00 mg/m³, 18.83 mg/m³ and 28,144 mg/m³ when Q was 1.00 and BT was 144.

Regression and ANOVA analyses for Air-to-Fuel Ratio

The model for the ratio of air to LPG (fuel) of gas burner emission characteristics is given in Equation 7 while the ANOVA analysis for the ratio of air to LPG (fuel) is presented in Table 8.

$$Air - fuel - Ratio = 11.0 - 19.1Q + 0.631BT + 18.9Q^2 + 0.000113BT^2 - 0.0683BT \quad (5)$$

Table 6: ANOVA table for TSP emission model prediction

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	1379157691	275831538	0.91	0.518
Q	1	185756	185756	0.00	0.981
BT	1	110044492	110044492	0.36	0.563
Q*Q	1	338762	338762	0.00	0.974
BT*BT	1	357571973	357571973	1.17	0.307
Q*BT	1	677488	677488	0.00	0.963

Error	9	2742323484	304702609		
Total	14	4121481174			
Model Summary:					
S	R-sq	R-sq(adj)	R-sq(pred)		
17455.7	33.46%	0.00%	0.00%		
Coefficients:					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	114017	881137	0.13	0.900	
Q	-48064	1946623	-0.02	0.981	932.71
BT	-1004	1670	-0.60	0.563	211.00
Q*Q	35924	1077391	0.03	0.974	926.71
BT*BT	4.50	4.15	1.08	0.307	49.00
Q*BT	-77	1626	-0.05	0.963	169.00

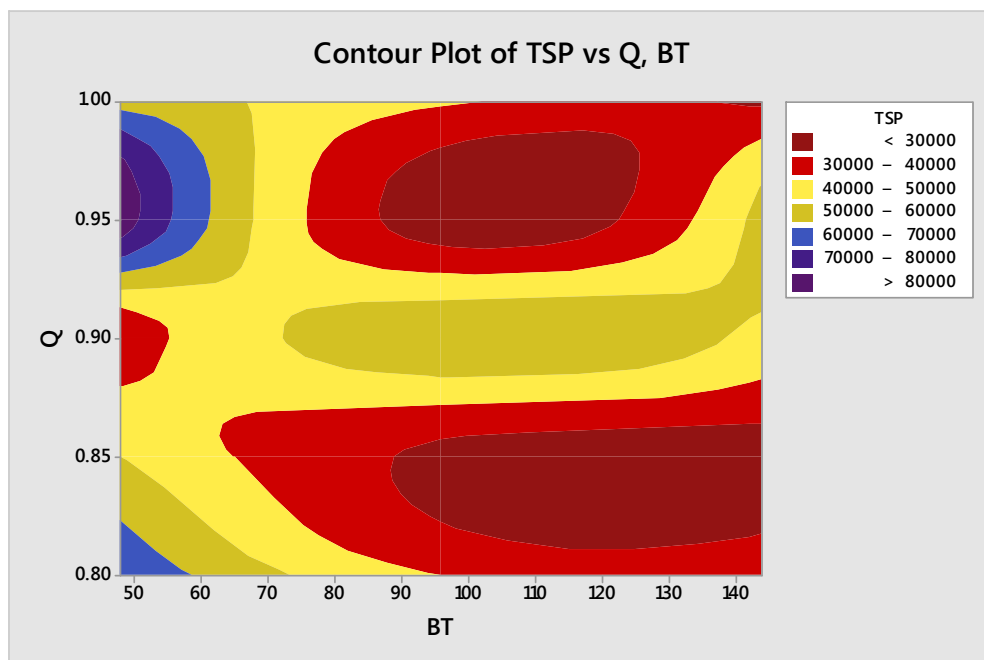


Figure 7: Contour plot for TSP emission prediction

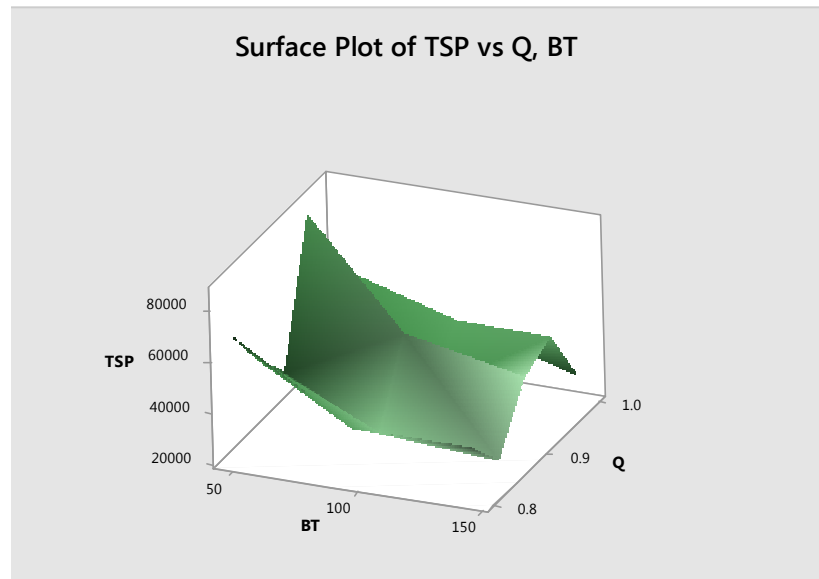


Figure 8: Surface plot for TSP emission prediction.

Table 7: Model prediction data for gas burner emission characteristics.

Q	BT	CO	NO _x	TSP
0.80	48	34.43	19.09	69091
0.80	96	145.20	16.34	40000
0.80	144	135.82	19.71	40000
0.85	48	42.17	23.60	50000
0.85	96	44.47	19.77	28000
0.85	144	161.14	24.47	25000
0.90	48	48.50	15.83	34286
0.90	96	39.60	20.41	57143
0.90	144	269.45	31.96	47293
0.95	48	351.82	25.68	85714
0.95	96	43.85	20.37	25455
0.95	144	271.03	18.83	53333
1.00	48	204.69	20.52	55385
1.00	96	46.41	17.57	41739
1.00	144	212.00	18.83	28144

Table 8: ANOVA table for air-to-fuel ratio model prediction.

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	18.4646	3.69293	21.43	0.000
Q	1	0.0294	0.02943	0.17	0.689
BT	1	0.4351	0.43511	2.53	0.146
Q*Q	1	0.0933	0.09334	0.54	0.480
BT*BT	1	0.2253	0.22533	1.31	0.282
Q*BT	1	0.5379	0.53792	3.12	0.111
Error	9	1.5507	0.17230		
Total	14	20.0153			
Model Summary:					
S	R-sq	R-sq(adj)	R-sq(pred)		
0.415088	92.25%	87.95%	77.43%		
Coefficients:					
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	11.0	21.0	0.52	0.614	
Q	-19.1	46.3	-0.41	0.689	932.71
BT	0.0631	0.0397	1.59	0.146	211.00
Q*Q	18.9	25.6	0.74	0.480	926.71
BT*BT	0.000113	0.000099	1.14	0.282	49.00
Q*BT	-0.0683	0.0387	-1.77	0.111	169.00

From Table 7, it can be seen that p-values are greater than 0.05 significant level thus, Q and BT have significant effect on the air-to-fuel balance of the pyrolysis process using the model. The low value of standard error (S) obtained as 0.415088 implies that the regression model effectively described air-to-fuel ratio response variable. Also, high R-sqvalue (92.25%) showed that the developed regression model for air-to-fuel ratio fits experimental data in TABLE 1. Values of predicted R-sq and adjusted R-sq obtained as 77.43% and 87.95% respectively, indicate that the regression model can predict response variable (air-to-fuel ratio) for new data and it can explain 87.95% variance in the response variable. The contour and surface plots for air-to-fuel ratio prediction model are shown in Figures 9 and 10 below.

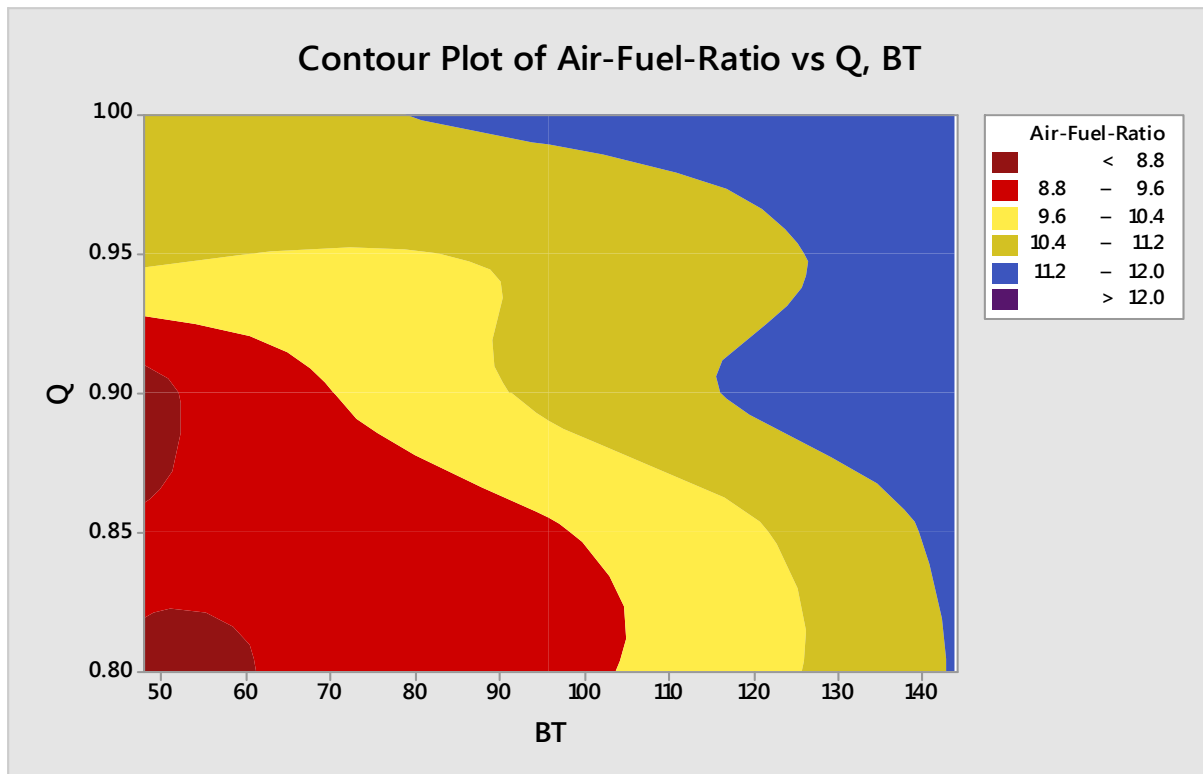


Figure 9: Contour plot for Air-to-Fuel ratio prediction

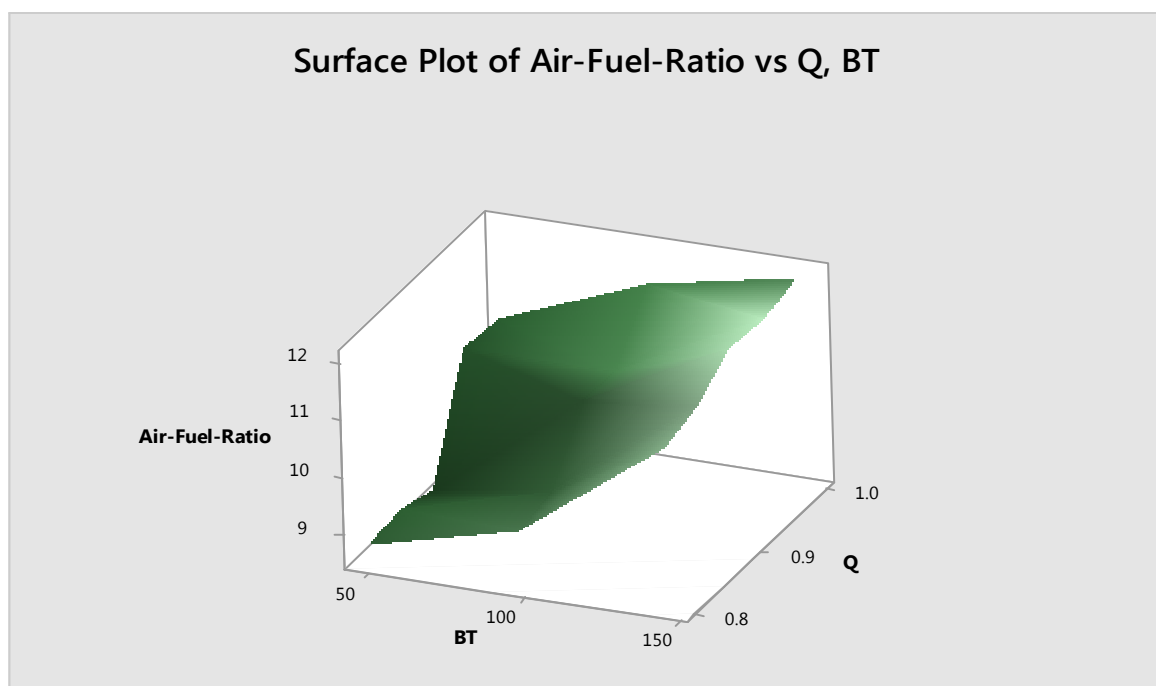


Figure 10: Surface plot for Air-to-Fuel ratio prediction.

From the contour plot of Figure 9, it can be seen that the pyrolysis process utilized a low (< 8.8) air-to-fuel balance according to the regression model for air-to-fuel ratio prediction, with a brown contour line, at the beginning of the experiment when Q was 0.8 litre/min and BT was < 50 but this ratio increased (between 9.6 and 10.4) with yellow contour line, during the experiment when Q was 0.9 litre/min and BT was < 100 . The highest ratio (between 11.2 and 12.0) with blue contour line was obtained at the end of experiment when Q was 1.0 and BT was > 140 . The surface plot of Figure 10, showed that air-to-Fuel ratio was highest at values > 11.0 when Q was < 1.0 litre/min and burner hole type was < 150 .

IV. Conclusion

Mathematical modeling of two phases of combustion process in the furnace of a gas-fired pyrolysis reactor was carried out in order to determine the optimum burner hole and air-to-fuel ratio for the improvement of the design of air-fuel intake port of the gas-fired pyrolysis reactor. The model was based on the data generated from the laboratory study of thermal efficiencies and air pollution impacts of locally fabricated liquefied petroleum gas (LPG) burners. For phase 1, model predicted data for thermal efficiency showed that thermal efficiency was highest at 69% when the gas flowrate and burner hole type were high at 1.0 litre/min and 144 respectively. High value (99.97%) of correlation coefficient from the regression analysis for thermal efficiency prediction indicated that the efficiency model is a good fit for experimental data and significant for predicting efficiency response variable. Correlation coefficient values for the characteristic emissions of the gas burners (phase 2), including CO, NO_x and TSP emissions are 50.35%, 22.76% and 39.20% respectively; these represent various fits of individual model equations to experimental data. Predicted data from each emission model showed that emissions are lowest at 212.00 mg/m³, 18.83 mg/m³, and 28,144.00 mg/m³ for CO, NO_x and TSP respectively; when Q and BT are highest at 1.0 and 144 respectively. Thus, effect of the pyrolysis process on the environment is minimal when operating at high gas flowrate and burner hole type. Regression model for air-to-fuel ratio gave a good predictive strength and fitness to experimental data with R-sq value of 92.25%.

Highest ratio (> 11.0) was predicted at high value of Q and BT values given as 1.0 litre/min and < 150 respectively. It can be concluded from the model results of the two phases of combustion process in the furnace of a gas-fired pyrolysis reactor that optimum thermal efficiency, air-to-fuel ratio and lowest emissions were predicted when burner hole type and gas flowrate were optimal at 144 and 1.0 litre/min.

References

- [1.] Zheleva, I., et al. *Mathematical Modeling of the Heat Transfer during Pyrolysis Process Used for End-of-Life Tires Treatment*. in *AIP Conference Proceedings* 1895, 030008 (2017); <https://doi.org/10.1063/1.5007367>. 2017.
- [2.] Gungor, C., et al., *Engine performance and emission characteristics of plastic oil produced from waste polyethylene and its blends with diesel fuel*. *International Journal of Green Energy*, 2015. **12**(1): p. 98-105.
- [3.] Tsai, W.T., M.K. Lee, and Y.M. Chang, *Fast pyrolysis of rice husk: Product yields and compositions*. *Bioresour Technol.*, 2007. **98**(1): p. 22–28.
- [4.] Kalargaris, S.G.I. and G. Tian, *Combustion, performance and emission analysis of a DI diesel engine using plastic pyrolysis oil*. *Fuel Process Technol* 2017. **157**: p. 108–115.
- [5.] A, Z.A. and G. Stavropoulos, *Pyrolysis of used automobile tires and residual char utilization*. *J Anal Appl Pyrolysis*, 2003. **70**: p. 711–722.
- [6.] Akinbomi, J. and M. Salami, *Generator Exhausts Control in Nigeria using Activated Carbon from Discarded Rubber Tyres*. *International Journal of Engineering Research & Technology*, 2022. **11**(2): p. 100-107.
- [7.] Aylón, E., et al., *Valorisation of waste tyre by pyrolysis in a moving bed reactor*. *Waste*

Manag, 2010. **30**: p. 1220–1224.

- [8.] Bandyopadhyay, S., et al., *An overview of rubber recycling*. Progress in Rubber, Plastics and Recycling Technology, 2008. **24**(2): p. 73–112.
- [9.] Akinbomi, J.G., et al., *Evaluation of Carbon Black Usage in Shoe Polish Production*. International Journal of Engineering Research & Technology, 2022. **11**(1): p. 491-495.
- [10.] Akinbomi, J.G., et al., eds. *Asphalt Making Potential of Pyrolytic Bitumen from Waste Rubber Tyres: An Adaptive Measure to Climate Change*. Handbook of Climate Change Resilience, ed. W. Leal-Filho. 2019, Springer: Cham.
- [11.] Zhang, O., et al., *Review of biomass pyrolysis oil properties and upgrading research*. Energy Conversion Management Journal,, 2007. **48**(1): p. 87-92.
- [12.] Srivastava, V.K., R.K. Jalan, and B. Sushil, *Development of Mathematical Model for the Prediction of Concentration in the Pyrolysis of Biomass Material*. Indian Journal of Chemical Technology, 1996. **3**: p. 71-76.
- [13.] Akinbomi, J.G., et al., *Design improvement of a gas-fired pyrolysis reactor*. Engineering & Technology Research Journal, 2021. **6** (1): p. 22-29.

Appendix

Acronyms and abbreviations

DF = degree of freedom

Adj SS = Adjusted sum of squares

Adj MS = Adjusted mean squares

F-value = Fischer value

P value = Probability value

R-sq = Correlation coefficient

S = significant level

CO = carbon monoxide emission

NOx = Nitrogenous oxide emission

TSP = total suspended particle emission