An Analytical Study of Asymmetrical Preloaded Bolted Joints

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ABSTRACT : *The purpose of this technical note is to develop an understanding of the influence of asymmetrical geometry within preloaded bolted joints. Classical analysis methods are applied to the analysis of preloaded bolted joints that use asymmetrical bolt group patterns. Both a detailed analysis of asymmetrical joints, using classical beam theory, and a less detailed design analysis are considered. The detailed analysis method is extended, using Rotscher's pressure cone, and is suitable to produce calculated bolt loads that can be used in a fatigue analysis. The design analysis provides a quick method of establishing the structural integrity of the asymmetrical joint. The detailed analysis method can be applied to the structure being connected by the bolted joint and the welds connecting the structure to the joint flanges. The design method is also appropriate for application to sprung suspension systems. The methods presented are suitable for use in automated procedures of calculation, such as spread sheets, MathCAD ©, SMathSutdio ©, etc.*

KEYWORDS -*asymmetric bolted joint, preloaded bolted, bolt preload, bolt tension, multi bolt*

I. INTRODUCTION

Bolted joints are an extremely useful feature in mechanical engineering. They allow disassembly for maintenance and end of life disposal purposes. Bolted joints also allow complex assemblies to be made on site without the need for specialist processes such as welding, stress relieving, heat treatment or other post process activities. Preloading the bolts also produces a stiff joint, suitable for load bearing structures subjected to load reversals and also offer good fatigue resistance.

There are basically two, related, methods of analysing preloaded bolted joints. Firstly, a detailed analysis that is based on the theory of beams. Secondly, a design analysis method that considers each bolt, and a region of flange surrounding it, as acting together and performing as a stiff spring.

Preloading the bolts of the joint is intended to produce a uniform, or near uniform, contact pressure at the faying surface. This in turn makes the two flanges of the joint perform as if they were a single, solid, structure. The detailed analysis of a preloaded bolted joint assumes that the classical theory of bending of beams can be applied to an, effectively, single continuous member. The stresses that would be produced in this 'solid member' by the application of external loads act to change the contact pressure at the faying surface and to change the tensile stresses in the preloaded bolts. Any external out of plane bending will also introduce a bending stress component in each of the bolts

The design analysis treats each bolt, and a region of the flanges surrounding the bolt, as a spring. This type of design analysis is a simplification of the detailed, classical, analysis but does not require the calculation of section properties. However, this simplification of the analysis does not reflect the flexural stiffness of the joint and results in an over estimate of the axial loads acting at the area of the joint flange being controlled by each bolt, and hence, an overestimate of the minimum bolt preload required to maintain closure of the joint.

Both the detailed and the design methods of analysis consider that shear stresses on the joint are supported by friction at the joint face. This requires the bolt preload to include an additional component of clamping force to support the in-plane loads. This additional clamping force has to be equivalent to the maximum shear stress divided by the friction coefficient for the faying surface. Some joints use dowels, or other positive means of restraint, to assist in supporting shear loads. These should be considered as providing alignment and preventing slip, but not as the primary method of supporting in-plane, shear, loads.

II. NOMENCLATURE

A_b	Tensile area of each bolt
A_j	Total area of joint (Faying surface plus bolts)
d_b	Nominal bolt diameter
$D_{f.e}$	Projected diameter of Rotscher's pressure cone at faying surface
$F_{b(n)}$	Bolt load in bolt 'n'
$F_{br(n)}$	Bolt-related load for bolt 'n'
F_{dp}	Design preload
F_p	Preload in each bolt
F_z	External axial load in direction of 'z' axis
$I'_{p.max}$	Maximum second moment of area about a principal axis
$I'_{p.min}$	Minimum second moment of area about a principal axis
$I_{xx.j}$	Second moment of area of joint about 'x' axis
$I'_{xx.j}$	Second moment of area transposed about x'-axis
$I_{xy.j}$	Product moment of area of joint
$I'_{xy.j}$	Transposed product moment of area of joint
$I_{yy.j}$	Second moment of area of joint about 'y' axis
$I'_{yy.j}$	Second moment of area transposed about y'-axis
M_x	External moment acting about 'x' axis
M'_x	Transposed moment
M_y	External moment acting about 'y' axis
M'_y	Transposed moment
N_b	Number of bolts in joint
P_p	Pressure at faying surface, preload pressure
P_f	Pressure at faying surface when external loads are applied
$t_{f.min}$	Flange thickness, minimum of the two flanges
x	Coordinate in plane of joint face
x'	Transposed coordinate
$x_{(n)}$	Coordinate of bolt 'n'
$x'_{(n)}$	Transposed coordinate of bolt'n'
y	Coordinate in plane of joint face
y'	Transposed coordinate
$y_{(n)}$	Coordinate of bolt 'n'
$y'_{(n)}$	Transposed coordinate of bolt'n'
φ	Rotscher's pressure cone angle, half cone angle
θ	Angle of principal axis

III. DETAIL ANALYSIS OF ASYMMETRICAL JOINTS

It is quite common for asymmetrical joints to be defined in terms of a geometrical coordinate system that are not the principal axes. This is illustrated in Figure 1.

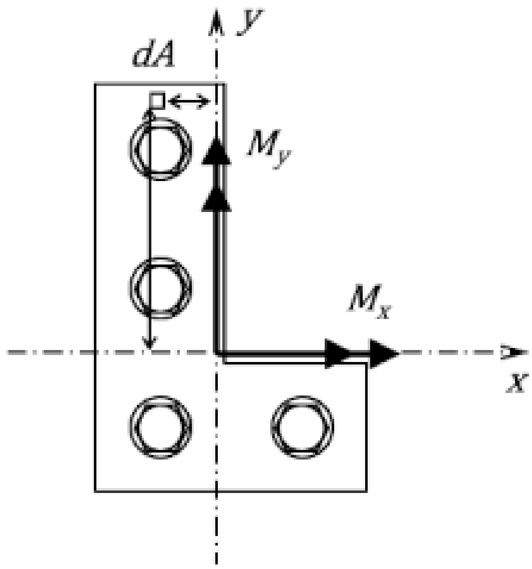

Figure 1. Asymmetrical Joint

The centroid of the joint lies at the point where $0 = \frac{1}{A_j}\int x \cdot dA$ and $0 = \frac{1}{A_j}\int y \cdot dA$

In Figure (1) the joints geometry and the moments M_x and M_y are defined with respect to the joint's coordinate system, which may not aligned with the principal axes of the joint. When dealing with an asymmetric joint it is necessary to determine the direction of the principal axes of the joint's cross-section. Then, in order to be able to carry out an analysis of the joint, the coordinate system and section properties have to be transposed to align with the joints principal axes, as illustrated in Figure (2). Also, the moments M_x and M_y have to be resolved to act about the principal axes.

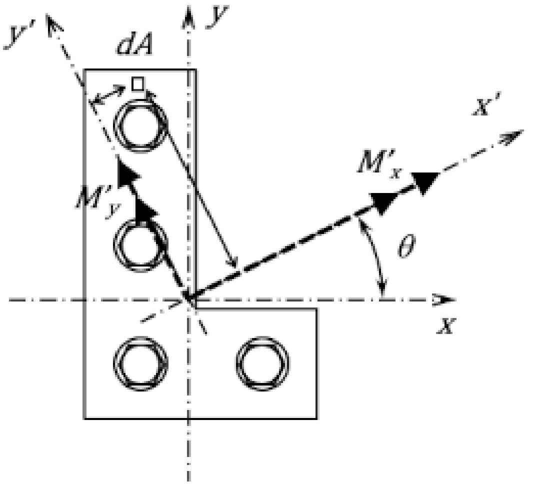

Figure 2. Transposed Coordinate System and Moments

If the principal axes are assumed to be at an angle θ to the axes defining the joint the transposed coordinate system is given by the following equations:

$$x' = x \cdot cos(\theta) + y \cdot sin(\theta) \qquad (1)$$

$$y' = y \cdot cos(\theta) - x \cdot sin(\theta) \qquad (2)$$

The second moments of area for the joint are described by the two equations:

$$I'_{xx.j} = \int y'^2 \cdot dA \qquad (3)$$

$$I'_{yy.j} = \int x'^2 \cdot dA \qquad (4)$$

Similarly, the product moment of area is described by the equation:

$$I'_{xy.j} = \int x' \cdot y' \cdot dA \qquad (5)$$

The principal axes are defined by when the product moment area, $I'_{xy.j}$, is zero. Hence, using equations (1) and (2) in equation (5):

$$I'_{xy.j} = \int \left(x \cdot cos(\theta) + y \cdot sin(\theta)\right) \cdot \left(y \cdot cos(\theta) - x \cdot sin(\theta)\right) \cdot dA$$

Working with this term:

$$I'_{xy.j} = \int x \cdot y \cdot \left(cos^2(\theta) - sin^2(\theta)\right) \cdot dA + \int (y^2 - x^2) \cdot \sin(\theta) \cdot \cos(\theta) \cdot dA$$

$$I'_{xy.j} = I_{xy.j} \cdot \left(cos^2(\theta) - sin^2(\theta)\right) + \left(I_{xx.j} - I_{yy}.j\right) \cdot \sin(\theta) \cdot \cos(\theta)$$

Resulting in the equation:

$$I'_{xy.j} = I_{xy.j} \cdot \cos(2 \cdot \theta) + \frac{1}{2} \cdot \left(I_{xx.j} - I_{yy.j}\right) \cdot \sin(2 \cdot \theta) \qquad (6)$$

Equating the product moment of area, $I'_{xy.j}$, to zero and rearranging the resulting equation shows that the angle of principal axis from the joints x-axis (positive anticlockwise) is given by the equation:

$$\theta = \frac{1}{2} \cdot arctan\left(\frac{2 \cdot I_{xy.j}}{I_{yy.j} - I_{xx.j}}\right) \qquad (7)$$

A term for the second moment of area about the x'-axis can be found by substituting equation (2) into equation (3):

$$I'_{xx.j} = \int \left(y \cdot cos(\theta) - x \cdot sin(\theta)\right)^2 \cdot dA$$

Working with this term:

$$I'_{xx.j} = \int \left(y^2 \cdot cos^2(\theta) + x^2 \cdot sin^2(\theta) - 2 \cdot x \cdot y \cdot sin(\theta) \cdot cos(\theta)\right) \cdot dA$$

Resulting in the equation:

$$I'_{xx.j} = I_{xx.j} \cdot cos^2(\theta) + I_{yy.j} \cdot sin^2(\theta) - 2 \cdot I_{xy.j} \cdot sin(\theta) \cdot cos(\theta) (8)$$

Similarly, a term for the second moment of area about the y'-axis can be found by substituting equation (1) into equation (4):

$$I'_{yy.j} = I_{yy.j} \cdot cos^2(\theta) + I_{xx.j} \cdot sin^2(\theta) + 2 \cdot I_{xy.j} \cdot sin(\theta) \cdot cos(\theta) (9)$$

It is possible to use equations (6), (8) and (9) to show that the maximum and minimum second moments of area about the principal axes are given by the following equations:

$$I'_{p.max} = \frac{1}{2} \cdot \left(\left(I_{xx.j} + I_{yy.j}\right) + \sqrt{\left(I_{yy.j} - I_{xx.j}\right)^2 + 4 \cdot I^2_{xy.j}}\right) \qquad (10)$$

$$I'_{p.min} = \frac{1}{2} \cdot \left(\left(I_{xx.j} + I_{yy.j}\right) - \sqrt{\left(I_{yy.j} - I_{xx.j}\right)^2 + 4 \cdot I_{xy.j}^2} \right) \quad (11)$$

It should be noted that the connection between, or the understanding of, which of the second moments of area given by equations (10) and (11) is parallel to the principal axis defined by the angle θ, obtained from equation (7) has been lost. This is usually relatively easy to establish by observation, although not by computation. There is an advantage in using equations (8) and (9) in preference to equations (10) and (11) when 'automatic' calculation methods, such as spreadsheets, MathCAD © and SMath Studio © are used since the relationships between second moments of area and direction of axes are maintained.

Besides transposing the joint coordinates and section properties, the analysis of the joint also requires the external out-of-plane moments to be transposed. The transposed moments are given by the following equations:

$$M'_x = M_x \cdot cos(\theta) + M_y \cdot sin(\theta) \quad (12)$$

$$M'_y = M_y \cdot cos(\theta) - M_x \cdot sin(\theta) \quad (13)$$

If it is assumed that the bolts are distributed in a regular manner across the faying surface and the centroid and principal axes of the bolt group coincide with those of the faying surface then the contact pressure at the faying surface under preload and the pressure distribution under the external loads can be given by the following two equations:

$$P_p = \frac{-1}{A_f} \cdot \sum_n F_b \quad (14)$$

$$P_f = P_p + \frac{F_z}{A_j} + \frac{M'_x}{I'_{xx.j}} \cdot y' - \frac{M'_y}{I'_{yy.j}} \cdot y' \quad (15)$$

Similarly, the total load on individual bolts in the joint can be given by the equation:

$$F_{b(n)} = F_p + \left(\frac{F_z}{A_j} + \frac{M'_x}{I'_{xx.j}} \cdot y'_{(n)} - \frac{M'_y}{I'_{yy.j}} \cdot x'_{(n)} \right) \cdot A_b \quad (16)$$

The negative sign in equation (14) indicates that the contact pressure at the faying surface is compressive. For the joint to be able to function correctly it is required that the contact pressure calculated by equation (15) remains compressive (i.e. negative) under all cases of external loading. This has to be true for all points on the faying surface. If in-plane external loads are applied, it is a requirement that the resultant shear stress at the joint does not overcome the friction between the joint's flanges. The analysis of shear loads on the joint is outside the scope of this paper but it has been discussed in detail by Welch (2018) in reference [1]. It is also a requirement that the total bolt load given by equation (16) does not exceed the proof load for the bolt. If the bolt load does exceed the proof load there could be some relaxation of the bolt preload, which in turn could lead to joint failure.

IV. Rotscher's Pressure Cone

The preceding work assumes a uniform, or near uniform, pressure distribution at the faying surface. In practice, the contact pressure of a preloaded joint will not be uniform across the faying surface. It has been shown by Rotscher (1927), reference [2], that each preloaded bolt influences an approximately circular region of the faying surface that surrounds it. As a result, the total effective area and the effective second moment of area of the joint are less than those of the nominal faying surface area and the nominal second moment of area. Hence, the surface contact pressure produced when the bolts are installed is higher than that predicted using the full, or nominal, faying surface area. Similarly, the change in bolt stress and the change in contact pressure due to applied loads are also higher. These two effects tend act to act cancel each other out when considering the external loads that could cause joint separation.

When considering a joint similar to that illustrated in Figure 3, where the bolts are not regularly distributed and the principal axes of the faying surface and bolt group are not coincident, the effects of Rotscher's pressure cone need to be taken into account.

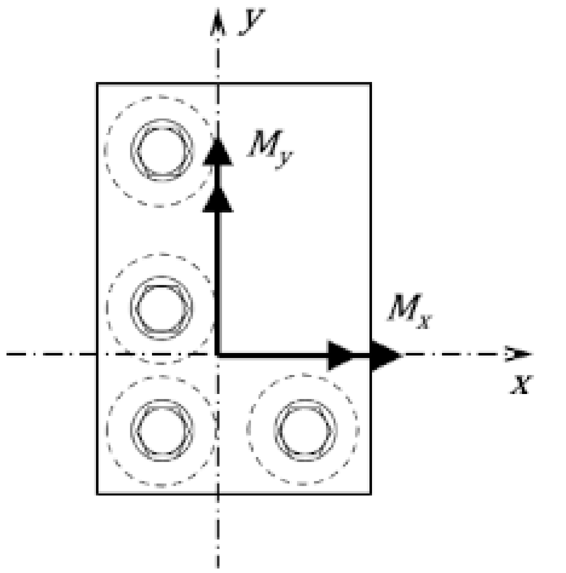

Figure 3. Joint with Non-regularly Distributed Bolts.

Figure 4 illustrates how Rotscher's pressure cone is formed and equation (17) provides an estimate of the resulting contact area diameter.

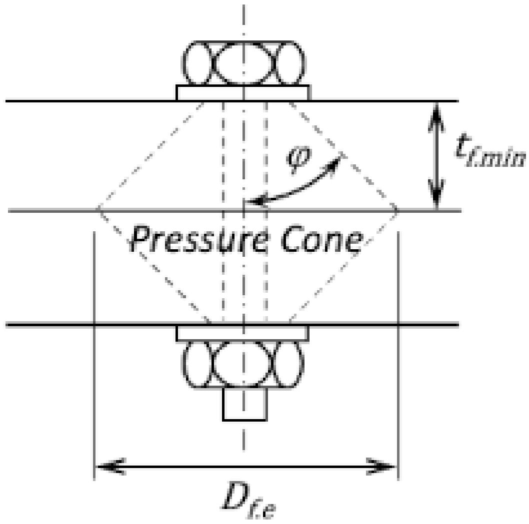

Figure 4. Rotscher's Pressure Cone.

$$D_{f.e} = 1.5 \cdot d_b + 2 \cdot t_{f.min} \cdot tan(\varphi) \quad (17)$$

Rotscher proposed a half cone angle of $\varphi = 45^o$. However, later researchers have found that this is an overestimate and the half cone angle, or pressure angle, can depend on a number of factors including flange thickness and the stiffening effects of surrounding structure attached to the flanges. A more realistic, or accurate, suggestion would be to use a pressure angle of $\varphi = 30^o$. Rotscher's pressure cone is discussed in more detail in section 8-5 *"Joints – Member Stiffness"* of reference [3] *"Shigley's Mechanical Engineering Design"* (2006). The application of a compression cone, based on Rotscher's pressure cone, is also discussed in detail in section 3 "*Load and deformation conditions*", and section 5 "*Calculation quantities*", of reference [4] *"Systematic calculation of highly stressed bolted joints Multi bolted joints"* (2014).

V. Design Analysis

The method of detailed analysis that has been described is particularly relevant as part of a full fatigue assessment or when investigating specific aspects of a bolted joint such as an in-service failure. In most instances the method of design analysis described by Welch (2019) in reference [5] "*A Paradigm for the Analysis of Preloaded Bolted Joints*" is adequate to provide evidence of structural integrity for a bolted joint. This design analysis is based on the assumption that each bolt assembly, comprising the nut, bolt washers and a region of the flanges defined by Rotscher's pressure cone, can be considered as a spring. Section 3.2 *"Principles for calculating single-bolted joints; analysis of forces and deformation*" of reference [4] *"Systematic calculation of highly stressed bolted joints Multi bolted joints"* (2014) presents a theoretical study of a single bolt assembly which shows the principle that forms the basis for this assumption.

In a design analysis, when only the location of the bolt centres is being considered, not the joints section properties, the centroid of the bolt group is defined as the point where $0 = \frac{1}{N_b} \cdot \sum_n x_{(n)}$ and $0 = \frac{1}{N_b} \cdot \sum_n y_{(n)}$

It is worth noting that the design analysis does not require the diameter of Rotscher's pressure cone or the effective spring stiffness to be calculated. The design analysis method is simply based on an understanding of how the effective spring stiffness influences the joint.

The direction of the principal axes of the bolt group can then be given by the following modified version of equation (7):

$$\theta = \frac{1}{2} \cdot arctan\left(\frac{2 \cdot \sum_n x_{(n)} \cdot y_{(n)}}{\sum_n x_{(n)}^2 - \sum_n y_{(n)}^2}\right) \quad (18)$$

The design analysis method also requires that the bolt centre coordinates are transposed into the coordinate system defined by the principal axes of the bolt group. This can be achieved by applying equation (1) and (2) to the bolt geometry. Re-writing these two equations in terms of the bolt location geometry:

$$x'_{(n)} = x_{(n)} \cdot cos(\theta) + y_{(n)} \cdot sin(\theta) \quad (19)$$

$$y'_{(n)} = y_{(n)} \cdot cos(\theta) - x_{(n)} \cdot sin(\theta) \quad (20)$$

The design analysis also requires the external out-of-plane moments to be transposed. The transposed moments are again given by equations (12) and (13), using the angle of direction of the principal axes as given by equation (18).

The bolt related load is then given by the following equation:

$$F_{b_{r(n)}} = \frac{F_z}{N_b} + \frac{M'_x}{\sum_n {y'}^2_{(n)}} \cdot y'_{(n)} - \frac{M'_y}{\sum_n {x'}^2_{(n)}} \cdot x'_{(n)} \quad (21)$$

The bolt related load given by equation (21) is not the load on an individual bolt; it is the component of external load that is passing through the region of the joint that is controlled by the bolt. In effect, the bolt related load represents an approximation to the minimum bolt preload required to ensure contact pressure is maintained across the faying surface.

This minimum bolt preload requirement represents a design preload. Section 3.8 of British Standard BS 7608:1990, "*Code of practice for Fatigue design and assessment of steel structures*" (1990), reference [6], says that the target or nominal bolt preload, F_p, should be at least 1.5 times the design preload, F_{dp}. Hence, the design requirement for the joint is:

$$F_{br_{(n)}} \leq F_{dp} \quad (22)$$

Where the design preload, F_{dp}, is given by:

$$F_{dp} = \frac{2}{3} \cdot F_p \quad (23)$$

The target preload, F_p, is usually based on a percentage of the bolt's proof load. Typically, calculations would assume a bolt preload of between 60% and 80% of the bolt proof load. When bolts are preloaded by tightening with a torque wrench, they are tightened to a specify bolt/nut 'make up' torque, which has been calculated or experimentally shown to achieve the required preload.

VI. CONCLUSION

The method of detailed analysis for an asymmetrical bolted joint is based on the theory of beams. Hence, the equations that are derived for this method of analysis can also be applied to the bending analysis of any asymmetrical structures that are being connected by the bolted joint.

Similarly, any welds attaching structures to the joint flanges can be analysed using the same principles.

The joint design should, ideally, produce the situation where the centroid and principle axes of the bolt group, load bearing structures being connected and any welds attaching the structures to the joint flanges are all aligned.

The design analysis method is a simplification of the detailed analysis and does not require the calculation of section properties. This simplification of the analysis does not account for the flexural stiffness of the joint and results in an over estimate of the axial loads acting at the area of the joint flange being controlled by each bolt.

The method of design analysis is based on the assumption that each bolt in the joint and a surrounding region of the flanges can be considered as acting as a spring. Therefore, this method of analysis can be applied to calculate load the distributions of sprung suspension systems, and hence calculate the spring deflections.

The analysis methods presented are suitable for use in 'automated' calculation procedures.

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