Some Properties of Soft ij-Delta Open Set In Soft Bitopology

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Abstract. For dealing with uncertainty researchers introduced the concept of soft set. In (2014) B. M. Ittanagi [10] defined several basic notions on soft bitopology and they studied many properties of them. In this paper, we introduce and study soft ij-regular open set and soft *ij*-regular closed set in soft bitopological space. Also, we present for the theory new definitions, characterizations, and result concerning soft bitopological space from where a soft*ij*-*δ*-open set, a soft *ij*-*δ*-closed set, soft *ij*-*δ*-interioe, soft *ij*-*δ*-clusore, soft *ij*-*δ*-derived set, soft *ij*-*δ*-border set, soft *ij*-*δ*-frontier set, and soft *ij*-*δ*-exterior set

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I. INTRODUCTION

The notion of δ-open sets in bitopological spaces was introduced by Banerjee [4], who used this notion to study δ-continuous functions in bitopological spaces. In (2014) B. M. Ittanagi [10] Introduced and studied the concept of soft bitopological spaces which are defined over an initial universe set with a fixed set of parameters.

The main purpose of this paper is to continue investigating soft sets in soft bitopological. It is organized as follows . In section 2 we write basic notions and results concerning the theory of soft sets, soft topological spaces, and soft bitopological. Section 3 contains the definition and some properties of soft *ij*-regular open sets and soft *ij*-regular closed sets on soft bitopology. In section 4, the notions of soft *ij*-*δ*-open set, soft *ij*-*δ*-closed set and soft *ij*-*δ*-bitopological space are defined and some of their properties are studied. Also, some other characterizations of soft *ij*-*δ*-interior and soft *ij*-*δ*-closure are investigated. Finally, in section 5, the definition and properties of soft *ij*-*δ*-derived set, soft *ij*-*δ*-border set, soft *ij*-*δ*-frontier set, and Soft *ij*-*δ*-exterior set are presented.

2- PRELIMINARIES

Definition 2.1. [13]. Let ***X*** be an initial universe set, ***P(X)*** the power set of X, that is the set of all subsets of ***X***, and ***E*** a set of parameters. A pair *(F,E)*, where *F* is a map from *E* to ***P(X)***, is called a soft set over ***X***. In what follows by ***SS(X,E)*** we denote the family of all soft sets over ***X***.

Definition 2.2. [13]. Let (F,E), (G,E)∈***SS(X,E)***. We say that the pair (F,E) is a soft subset of *(G,E)* if F(p)⊆G(p), for every p∈***E***. Symbolically, we write (F,E)⊑(G,E*)*. Also, we say that the pairs *(F,E)* and (G,E) are soft equal if (F,E)⊑ (G,E) and (G,E)⊑(F,E), Symbolically, we write (F,E)=(G,E).

Definition 2.3. [13]. Let ***I*** be an arbitrary index set and$\{(F_i,E):i\in I\}$∈***SS\X,E)***. Then.

(1) The soft union of these soft sets is the soft set (F,E)∈***SS(X,E)***, where the map F:E→P(X) is defined as follows: $F(p)=\cup\{F_i(p):i\in I\}$, for every p∈E. Symbolically, we write $(F,E)=\sqcup\{(F_i,E):i\in I\}$.

(2) The soft intersection of these soft sets is the soft set (F,E)∈***SS(X,E)***, where the map F:E→P(X) is defined as follows: $F(p)=\cap\{F_i(p):i\in I\}$, for every p∈E. Symbolically, we write $(F,E)=\sqcap\{(F_i,E):i\in I\}$.

Definition 2.4. [19]. Let (F,E)∈***SS(X,E)***. The soft complement of (F,E) is the soft set (H,E)∈***SS(X,E)***, where the map H:E→P(X) defined as follows: H(p)=X\F(p), for every p∈E. Symbolically, we write $(H,E)=(F,E)^c$. obviously, $(F,E)^c=(F^c,E)$ [10]. For two given subsets (M,E), (N,E)∈***SS(X,E)*** [28], we have.

(1) $((M,E)\sqcup(N,E))^c=(M^c,E)\sqcap(N^c,E)$,

(2) $((M,E)\sqcap(N,E))^c=(M^c,E)\sqcup(N^c,E)$.

Definition 2.5. [13]. The soft set (F,E)∈***SS(X,E)***, where F(p)=ϕ, for every p∈E is called the A-null soft set of ***SS(X,E)*** and denoted by $\mathbf{0_E}$. The soft set (F,E)∈***SS(X,E)***, where F(p)=X, for every p∈E is called the E-absolute soft set of ***SS(X,E)*** and denoted by $\mathbf{1_E}$.

Definition 2.6. [19]. The soft set (F,E)∈***SS(X,E)*** is called a soft point in X, denoted by e_F, if for the element e∈E, F(e)≠ϕ and F(e′)=ϕ for all e′∈E\{e}.

Definition 2.7. [19]. The soft point e_F is said to be in the soft set (G,E), denoted by $e_F\tilde{\in}(G,E)$, if for the element e∈E, F(e)⊑G(e).

Definition 2.8. [9]. Two soft points e_G, e_H in ***SP(X)*** are distinct, written $e_G\neq e_H$, if there corresponding soft sets (G,E) and (H,E) are disjoint.

Proposition 2.9. [19]. Let e_F∈***SP(X)*** and (G,E)∈***SS(X,E)***. If $e_F\tilde{\in}(G,E)$, then $e_F\tilde{\notin}(G^c,E)$.

Definition 2.10. [19]. Let ***X*** be an initial universe set, ***E*** a set of parameters and $\tilde{\tau}\subseteq$***SS(X,E)***. We say that the family $\tilde{\tau}$ defines a soft topology on ***X*** if the following axioms are true.

(1) $\mathbf{0_E},\mathbf{1_E}\in\tilde{\tau}$.

(2) If (G,E), (H,E)$\in\tilde{\tau}$, then (G,E)⊓(H,E)$\in\tilde{\tau}$.

(3) If $(G_i,E)\in\tilde{\tau}$ for every i∈***I***, then $\sqcup\{(G_i,E):i\in I\}\in\tilde{\tau}$.

The triplet ***(X,τ̃,E)*** is called a soft topological space. The members of τ are called soft open sets in ***X***. Also, a soft set (F,E) is called soft closed set if the complement (F^c,E) belongs to τ. The family of all soft closed sets is denoted by $\tilde{\tau}^c$.

Definition 2.11. Let ***(X,τ̃,E)*** be a soft topological space and (F,E)∈***SS(X,E)***:

(1) The soft closure of (F,E) [17] is the soft set Cls(F,E)=⊓{(S,E):(S,E)∈τ^{c}, (F,E)⊑(S,E)}.

(2) The soft interior of (F,E) [19] is the soft set Ints(F,E)=$\sqcup${(S,E):(S,E)$\in$τ, (S,E)$\sqsubseteq$(F,E)}.

Theorem 2.1. [8]. Let ***(X,$\tilde{\tau}$,E)*** be a soft topological space over ***X*** and(F,E), (G,E)$\in$***SS(X,E)***. The following statements are true.

(1)Cl_s($\mathbf{0_E}$)= $\mathbf{0_E}$ andCl_s($\mathbf{1_E}$)= $\mathbf{1_E}$.

(2) (F,E)$\sqsubseteq$$Cl_s$(F,E).

(3) (F,E) is a soft closed set if and only ifCl_s(F,E)=(F,E).

(4)Cl_s(Cl_s(F,E))=Cl_s(F,E).

(5) (F,E)$\sqsubseteq$(G,E) impliesCl_s(F,E)$\sqsubseteq$$Cl_s$(G,E).

(6)Cl_s((F,E)$\sqcup$(G,E))=Cl_s(F,E)$\sqcup$$Cl_s$(G,E).

(7)Cl_s((F,E)$\sqcap$(G,E))$\sqsubseteq$$Cl_s(F,E)\sqcap$$Cl_s$(G,E).

Theorem 2.2. [8]. Let ***(X,$\tilde{\tau}$,E)*** be a soft topological space over ***X*** and (F,E), (G,E)$\in$***SS(X,E)***. The following statements are true.

(1) Int_s($\mathbf{0_E}$)= $\mathbf{0_E}$ and Int_s($\mathbf{1_E}$)= $\mathbf{1_E}$.

(2) Int_s(F,E)$\sqsubseteq$(F,E).

(3) Int_s(Int_s(F,E))=Int_s(F,E).

(4) (F,E) is a soft open set if and only if Int_s(F,E)=(F,E).

(5) (F,E)$\sqsubseteq$(G,E) implies Int_s(F,E)$\sqsubseteq$$Int_s$(G,E).

(6) Int_s(F,E)$\sqcap$$Int_s$(G,E)=$Int_s$((F,E)$\sqcap$(G,E)).

(7) Int_s(F,E)$\sqcup$$Int_s(G,E)\sqsubseteq$$Int_s$((F,E)$\sqcup$(G,E)).

Definition 2.12. [19]. A soft set (G,E) in a soft topological space ***(X,$\tilde{\tau}$,E)*** is called a soft neighborhood (briefly: ***snbd***) of a soft point $e_F$$\in$***SP(X)*** if there exists a soft open set (H,E) such that $e_F\tilde{\in}$(H,E)$\sqsubseteq$(G,E). The soft neighborhood system of a soft point e_F, is denoted by ***N$\tilde{\tau}$(eF)***, is the family of all of its soft neighborhoods.

Theorem 2.3. [19]. Let ***(X,$\tilde{\tau}$,E)*** be a soft topological space and (G,E)$\in$***SS(X,E)***. Then.

(1) (Cl_s(G,E))^c=Int_s(G^c,E).

(2) (Int_s(G,E))^c=Cl_s(G^c,E).

Definition 2.14. [18]. Let ***(X,$\tilde{\tau}$,E)*** be soft topological space and (F,E) be a soft set over ***X***.

(1) (F,E) is said to be a soft regular open set in X if (F,E)=Int_s(Cl_s(F,E), denoted by (F,E)$\in$***SRO(X,E)***.

(2) (F,E) is said to be a soft regular closed set in X if (F,E)=Cl_s(Int_s(F,E),denoted by (F,E)$\in$***SRC(X,E)***.

Remark 2.1. [18]. Every soft regular open set in soft topological space ***(X,$\tilde{\tau}$,E)*** is soft open set

Definition 2.15.[10] Let ***(X,$\tilde{\tau}_i$,E)*** and ***(X,$\tilde{\tau}_j$,E)*** be the two different soft topologies on ***X***. Then ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** is called a soft bitopological space

3- SOFT*ij-REGULAR* OPEN SETSAND *SOFT ij*-REGULARCLOESD SETS.

In this section, we define and introduce some properties of the soft ij-regular open sets and soft *ij*-regular closed sets.

Definition 3.1. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***be soft bitopological space and (F,E) be a soft set over X. Also i,j∈{1,2} and i≠j

(1) (F,E) is said to be a soft *ij*-regular open set in X if (F,E)=*i-*Int_s*(j-*Cl_s(F,E)), denoted by (F,E)∈***ij-SRO(X,E)***.

(2) (F,E) is said to be a soft *ij*-regular closed set in X if (F,E)=*i-*Cl_s*(j-*Int_s(F,E)),denoted by (F,E)∈***ij-SRC(X,E)***.

Remark 3.1.$\mathbf{0}_E$ and $\mathbf{1}_E$ are always soft *ij*-regular open set and soft *ij*-regular closed set.

Remark 3.2. Every soft *ij*-regular open set in soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** is soft open. but the converse is not true, which follows from the following example.

Example 3.1. Let *X*={h_1,h_2,h_3,h_4}, *E*={e_1,e_2} then $\tilde{\tau}_1$={$\mathbf{0}_E$,$\mathbf{1}_E$,(F_1,E),(F_2,E),(F_3,E),(F_4,E),(F_5,E),(F_6,E)}, and$\tilde{\tau}_2$={$\mathbf{0}_E$,$\mathbf{1}_E$,(G_1,E),(G_2,E),(G_3,E),(G_4,E),(G_5,E),(G_6,E)}, where

(F_1,E)={(e_1,{h_1}),(e_2,{h_1})},

(F_2,E)={(e_1,{h_2}),(e_2,{h_2})},

(F_3,E)={(e_1,{h_1,h_2}),(e_2,{h_1,h_2})},

(F_4,E)={(e_1,{h_2,h_3}),(e_2,{h_2,h_3})},

(F_5,E)={(e_1,{h_1,h_2,h_3}),(e_2,{h_1,h_2,h_3})},

(F_6,E)={(e_1,{h_1,h_2,h_4}),(e_2,{h_1,h_2,h_4})},

(G_1,E)={(e_1,{h_3}),(e_2,{h_3})},

(G_2,E)={(e_1,{h_4}),(e_2,{h_4})},

(G_3,E)={(e_1,{h_3,h_4}),(e_2,{h_3,h_4})},

(G_4,E)=(F_4,E),

(G_5,E)=(F_5,E),

(G_6,E)=(F_6,E).

Then (F_4,E), (F_5,E), (F_6,E)∈***12-SRO(X,E)***. And (F_1,E),(F_2,E),(F_3,E)$\tilde{\in}$ $\tilde{\tau}_1$, (G_1,E),(G_2,E),(G_3,E)$\tilde{\in}$ $\tilde{\tau}_2$, but the (F_1,E), (F_2,E), (F_3,E), (G_1,E), (G_2,E), (G_3,E)∉***12-SRO(X,E)***.

Theorem 3.1. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***be a soft bitopological space. Let $(F_1,E),(F_2,E)\in$ ***ij-SRO(X,E)***. Then. $(F_1,E)\sqcap(F_2,E)\in$ ***ij-SRO(X,E)***.

proof Let (F_1,E), $(F_2,E)\in$ ***ij-SRO(X,E)***. Such that $(F_1,E)=i\text{-}Int_s(j\text{-}Cl_s(F_1,E))$, $(F_2,E)=i\text{-}Int_s(j\text{-}Cl_s(F_2,E))$. Since $(F_1,E)\sqcap(F_2,E)\sqsubseteq j\text{-}Cl_s((F_1,E)\sqcap(F_2,E))$. Then $i\text{-}Int_s(F_1,E)\sqcap(F_2,E))\sqsubseteq i\text{-}Int_s(j\text{-}Cl_s((F_1,E)\sqcap(F_2,E)))$, therefore $(F_1,E)\sqcap(F_2,E))\sqsubseteq i\text{-}Int_s(j\text{-}Cl_s((F_1,E)\sqcap(F_2,E)))$.

Conversely, Since $j\text{-}Cl_s((F_1,E)\sqcap(F_2,E))\sqsubseteq j\text{-}Cl_s(F_1,E)\sqcap j\text{-}Cl_s(F_2,E)$. Then $i\text{-}Int_s(j\text{-}Cl_s((F_1,E)\sqcap(F_2,E)))\sqsubseteq i\text{-}Int_s(j\text{-}Cl_s(F_1,E))\sqcap i\text{-}Int_s(j\text{-}Cl_s(F_2,E))$. Imply that $i\text{-}Int_s(j\text{-}Cl_s((F_1,E)\sqcap(F_2,E)))\sqsubseteq(F_1,E)\sqcap(F_2,E)$, and we are done.

Remark 3.3. A soft set (F,E) in a soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***is soft *ij*-regular open set if and only if soft complement (F^c,E) is soft *ij*-regular closed set.

Corollary 3.1. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** be a soft bitopological space, and $(F_1,E),(F_2,E)\in$ ***ij-SRC(X,E)***. $(F_1,E)\sqcup(F_2,E)\in$ ***ij-SRC(X,E)***.

proof. The proof is similar on Theorem 3.5.

4.SOFT *ij-δ-OPEN SET* ANDSOFT*ij-δ*-CLOSED SET.

In this section, we define soft *ij-δ*-open set, soft *ij-δ*-closed set , soft *ij-δ*-interior set and, soft *ij-δ*-closure set and investigate some of their related properties.

Definition 4.1. A soft set (U,E) is soft *ij-δ*-open set if for each $e_F\tilde{\in}(U,E)$, there exists a soft *ij*-regular open set (G,E) such that $e_F\tilde{\in}(G,E)\sqsubseteq(U,E)$, denoted by $(U,E)\in$ ***ij-SδO(X,E)***. i.e. A soft set is soft *ij-δ*-open set if it is the union of soft *ij*-regular open sets.The complement of soft *ij-δ*-open set is said to be soft *ij-δ*-closed set, denoted by $(U^c,E)\in$ ***ij-SδC(X,E)***.

Proposition 4.1. The familly $\tilde{\tau}_{\delta ij}$of all soft *ij-δ*-open sets defines a soft bitopology on ***X***.

Proof. (1) It obvious that $\mathbf{0_E}$, $\mathbf{1_E}\tilde{\in}\ \tilde{\tau}_{\delta ij}$.

(2) Let(U_1,E), $(U_2,E)\tilde{\in}\ \tilde{\tau}_{\delta ij}$. We shall prove that $(U_1,E)\sqcap(U_2,E)\tilde{\in}\ \tilde{\tau}_{\delta ij}$. There exists (G_1,E), $(G_2,E)\in$ ***ij-SRO(X,E)*** sach that $(G_1,E)\sqsubseteq(U_2,E)$ and $(G_2,E)\sqsubseteq(U_2,E)$. Then $(G_1,E)\sqcap(G_2,E)\sqsubseteq(U_1,E)\sqcap(U_2,E)$. But $(G_1,E)\sqcap(G_2,E)\in$ ***ij-SRO(X,E),*** by Theorem 3.5. Then $(U_1,E)\sqcap(U_2,E)\tilde{\in}\ \tilde{\tau}_{\delta ij}$

(3) Let $\{(U_k,E):k\in\boldsymbol{I}\}\sqsubseteq\tilde{\tau}_{\delta ij}$. We shall prove that $\sqcup\{(U_k,E):k\in\boldsymbol{I}\}\in\tilde{\tau}_{\delta ij}$. Eash $\{(U_k,E):k\in\boldsymbol{I}\}$ is union of members of *ij*-regular open sets. Then $\sqcup\{(U_k,E):k\in\boldsymbol{I}\}$ is also is union of members of *ij*-regular open sets. Thus $\sqcup\{(U_k,E):k\in\boldsymbol{I}\}\in\tilde{\tau}_{\delta ij}$.

Example 4.1. The soft bitopological space is same in Example 3.1. We heve (F_3,E), (F_5,E), (F_6,E)∈***12-SRO(X,E)***. That we get $\tilde{\tau}_{\delta 12}$={$\mathbf{0_E}$, $\mathbf{1_E}$,(F_3,E),(F_5,E),(F_6,E)}.

Proposition 4.2. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** be a soft bitopological space.The family $\tilde{\tau}^c_{\delta ij}$. has the following propertie.

(1) $\mathbf{0_E}$, $\mathbf{1_E}$$\tilde{\in}$ $\tilde{\tau}^c_{\delta ij}$.

(2) If (V,E),(K,E)$\tilde{\in}$ $\tilde{\tau}^c_{\delta ij}$, then (V,E)⊔(K,E)$\tilde{\in}$ $\tilde{\tau}^c_{\delta ij}$.

(3) If $(V_k,E)$$\tilde{\in}$ $\tilde{\tau}^c_{\delta ij}$, for every k∈***I***, then ⊓{(V_k,E):k∈***I***}$\tilde{\in}$ $\tilde{\tau}^c_{\delta ij}$.

proof. The proof verify directly from the proposition 4.1.

Definition 4.2. A soft set (G,E) in a soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** is called a soft *ij-δ*-neighborhood (briefly: ***ij-δ-snbd)*** of a soft point $e_F$$\tilde{\in}$***SP(X)*** if there exists a soft *ij-δ*-open set (H,E) such that $e_F$$\tilde{\in}$(H,E)⊑(G,E). The soft ij-δ-neighborhood system of a soft point e_F , denoted by ***ij-N$\tilde{\tau}$δ(e_F)***, is the family of all of its soft ij-δ-neighborhoods.

Proposition 4.3. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** be soft bitopological space , (U,E)∈***SS(X,E)***.Then (U,E) is soft ij-δ-open set if only if (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)*** for every $e_F$$\tilde{\in}$(U,E).

Proof. Let (U,E)∈ ***ij-SδO(X,E)*** and $e_F$$\tilde{\in}$(U,E). Thus $e_F$$\tilde{\in}$(U,E)⊑(U,E). And then (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)*** for every $e_F$$\tilde{\in}$(U,E).

Conversely Let (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)*** for every $e_F$$\tilde{\in}$(U,E). there exists a soft ij-δ-open set (H,E) such that $e_F$$\tilde{\in}$(H,E)⊑(U,E). Therefore (U,E)=⊔{(H,E):$e_F$$\tilde{\in}$(U,E), (H,E)∈$\tilde{\tau}_{\delta ij}$}. Then (U,E)∈***ij-SδO(X,E)***.

Proposition 4.4. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** be soft bitopological space, (U,E), (G,E)∈***SS(X,E)***.Then.

(1) If (G,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***, Then $e_F$$\tilde{\in}$(G,E).

(2) If (G,E), (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)*** then (G,E)⊓(U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***.

(3) If (G,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***, and (G,E)⊑(U,E) then (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***.

proof. (1) Obvious .

(2) Pick (G,E), (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***. There exists (H_1,E), (H_2,E)∈***ij-SδO(X,E)***, such that $e_F$$\tilde{\in}$$(H_1,E)$⊑(G,E) and $e_F$$\tilde{\in}$$(H_2,E)$⊑(U,E). Then $e_F$$\tilde{\in}$$(H_1,E)$⊓$(H_2,E)$⊑(G,E)⊓(U,E). Thus (G,E)⊓(U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***.

(3) Let (G,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***, (G,E)⊑(U,E). There exists (H,E)∈ ***ij-SδO(X,E)***, such that $e_F$$\tilde{\in}$(H,E)⊑(G,E). Hence $e_F$$\tilde{\in}$(H,E)⊑(G,E)⊑(U,E). then (U,E)∈ ***ij-N$\tilde{\tau}$δ(e_F)***

Definition 4.3. Let (G,E) be a soft set of soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***.A soft point e_F is called a soft ij-δ-interior point of (G,E) if there exists a soft ij-δ-open set (U,E) such that $e_F$$\tilde{\in}$(U,E)⊑(G,E). The set of all soft ij-δ-interior points of (G,E) is called the soft *ij-δ*-interior of (G,E) and is denoted by *ij-*Int_s^{δ}(G,E).

Remark 4.1. Let (G,E) be a soft set of soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***. A soft ij-δ-interior of (G,E) in ***SS(X,E)*** is

ij-Int $_s{}^{\delta}$(G,E)=⊔$_{k\in I}$}{(H$_k$},E):(H$_k$,E)$\tilde{\in}$ $\tilde{\tau}_{\delta ij}$, (H$_k$,E)⊑(G,E)}.

Proposition 4.5. Let ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** be a soft bitopological space and (G,E), (U,E)∈***SS(X,E)***. Then the following statements are true.

(1) *ij-Int$_s{}^{\delta}$*(U,E)⊑*i-Int$_s$*(U,E), where *i-Int$_s$*(U,E) is the soft interior of (U,E) in ***(X,$\tilde{\tau}_i$,E)***.

(2) *ij-Int$_s{}^{\delta}$*(U,E) is the largest soft *ij-δ*-open set contained in (U,E).

(3) (U,E) is soft *ij-δ*-open set if and only if *ij-Int$_s{}^{\delta}$*(U,E)=(U,E).

(4) *ij-Int$_s{}^{\delta}$*(**0$_E$**)= **0$_E$** and *ij-Int$_s{}^{\delta}$*(**1$_E$**)= **1$_E$**

(5) *ij-Int$_s{}^{\delta}$*(*ij-Int$_s{}^{\delta}$*(U,E))=*ij-Int$_s{}^{\delta}$*(U,E).

(6) If (U,E)⊑(G,E), then *ij-Int$_s{}^{\delta}$*(U,E)⊑*ij-Int$_s{}^{\delta}$*(G,E).

(7) *ij-Int$_s{}^{\delta}$*(U,E)⊔*ij-Int$_s{}^{\delta}$*(G,E)⊑*ij-Int$_s{}^{\delta}$*((U,E)⊔(G,E)).

(8) *ij-Int$_s{}^{\delta}$*((U,E)⊓(G,E))=*ij-Int$_s{}^{\delta}$*(U,E)⊓*ij-Int$_s{}^{\delta}$*(G,E).

proof. (1) Obvious.

(2) By remark 4.10 aobve *ij-Int$_s{}^{\delta}$*(U,E)=⊔{(H$_k$,E):(H$_k$,E)$\tilde{\in}$ $\tilde{\tau}_{\delta ij}$, (H$_k$,E)⊑(U,E)}. Thus *ij-Int$_s{}^{\delta}$*(U,E) is a soft *ij-δ*-open set soft subset of (U,E). Now let (H,E)$\tilde{\in}$ $\tilde{\tau}_{\delta ij}$ and e$_F$$\tilde{\in}$(H,E) then e$_F$$\tilde{\in}$(H,E)⊑(U,E). Therefore e$_F$ is a soft *ij-δ*-interior point of (U,E). Thus e$_F$$\tilde{\in}$(H,E) ⊑*ij-Int$_s{}^{\delta}$*(U,E) which shows that every soft *ij-δ-* open soft subset of (U,E) is contained in *ij-Int$_s{}^{\delta}$*(U,E). Hence *ij-Int$_s{}^{\delta}$*(U,E) is the largest soft*ij-δ*-open set contained in (U,E).

(3) Let (U,E) is soft *ij-δ*-open set.Then a soft *ij-δ*-open set containing all of its soft ij-points, it follows that every soft ij-point of (U,E) is a soft *ij-δ*-interior of (U,E). Thus (U,E)⊑*ij-Int$_s{}^{\delta}$*(U,E). Let e$_F$$\tilde{\in}$*ij-Int$_s{}^{\delta}$*(U,E) there exists (H,E)$\tilde{\in}$ $\tilde{\tau}_{\delta ij}$ such that e$_F$$\tilde{\in}$(H,E)⊑(U,E) then *ij-Int$_s{}^{\delta}$*(U,E)⊑(U,E). Therefore *ij-Int$_s{}^{\delta}$*(U,E)=(U,E).

Conversely, since *ij-Int$_s{}^{\delta}$*(U,E)=(U,E), then (U,E)$\tilde{\in}$ $\tilde{\tau}_{\delta ij}$. Hence (U,E) is soft *ij-δ*-open set if and only if *ij-Int$_s{}^{\delta}$*(U,E)=(U,E).

(4) Is obvious.

(5) Sinsce *ij-Int$_s{}^{\delta}$*(U,E) is soft *ij-δ*-open set. We have *ij-Int$_s{}^{\delta}$*(*ij-Int$_s{}^{\delta}$*(U,E))=*ij-Int$_s{}^{\delta}$*(U,E).

(6) Let e$_F$$\tilde{\in}$*ij-Int$_s{}^{\delta}$*(U,E) there exists (H,E)$\tilde{\in}$ $\tilde{\tau}_{\delta ij}$ soft containing e$_F$ such that (H,E)⊑(U,E). But (U,E)⊑(G,E). Then (H,E)⊑(G,E). Which implies that e$_F$$\tilde{\in}$*ij-Int$_s{}^{\delta}$*(G,E). Thus *ij-Int$_s{}^{\delta}$*(U,E)⊑*ij-Int$_s{}^{\delta}$*(G,E).

(7) Since (U,E)⊑(U,E)⊔(G,E). We have *ij-Int$_s{}^{\delta}$*(U,E)⊑*ij-Int$_s{}^{\delta}$*((U,E)⊔(G,E)), and (G,E)⊑(U,E)⊔(G,E).Then *ij-Int$_s{}^{\delta}$*(G,E)⊑*ij-Int$_s{}^{\delta}$*((U,E)⊔(G,E)). Therefore *ij-Int$_s{}^{\delta}$*(U,E)⊔*ij-Int$_s{}^{\delta}$*(G,E)⊑*ij-Int$_s{}^{\delta}$*((U,E)⊔(G,E)).

(8) Since (U,E)⊓(G,E)⊑(U,E). Then *ij-Int*$_s^\delta$((U,E)⊓(G,E))⊑*ij-Int*$_s^\delta$(U,E). Also *ij-Int*$_s^\delta$((U,E)⊓(G,E))⊑*ij-Int*$_s^\delta$(G,E). We get *ij-Int*$_s^\delta$((U,E)⊓(G,E))⊑*ij-Int*$_s^\delta$(U,E)⊓*ij-Int*$_s^\delta$(G,E).

Conversely, by the definition of a soft *ij-δ*-interior, *ij-Int*$_s^\delta$(U,E)⊑(H,E) and *ij-Int*$_s^\delta$(G,E)⊑(G,E). Then *ij-Int*$_s^\delta$(U,E)⊓*ij-Int*$_s^\delta$(G,E)⊑(U,E)⊓(G,E). Since *ij-Int*$_s^\delta$((U,E)⊓(G,E)) is the biggest soft *ij-δ*-open set that is contained by (U,E)⊓(G,E).. Hence *ij-Int*$_s^\delta$(U,E)⊓*ij-Int*$_s^\delta$(G,E)⊑*ij-Int*$_s^\delta$((U,E)⊓(G,E)). Therefor *ij-Int*$_s^\delta$((U,E)⊓(G,E))=*ij-Int*$_s^\delta$(U,E)⊓*ij-Int*$_s^\delta$(G,E).

Example 4.2. The soft bitopological space ***(X,*$\tilde{\tau}_1$*,*$\tilde{\tau}_2$*,E)*** is the same as in Example 3.1, and Example 4.1 Suppose that (U,E)={(e_1,{h_2}),(e_2,{h_2})} and (G,E)={(e_1,{h_3}),(e_1,{h_3})}. We get *12-Int*$_s^\delta$(U,E)⊔*12-Int*$_s^\delta$(G,E)= $\mathbf{0_E}$, and *12-Int*$_s^\delta$((U,E)⊔(G,E))={(e_1,{h_2,h_3}),(e_1,{h_2,h_3})}. One can deduce that *12-Int*$_s^\delta$(U,E)⊔*12-Int*$_s^\delta$(G,E)⊑*12-Int*$_s^\delta$((U,E)⊔(G,E)), and *12-Int*$_s^\delta$((U,E)⊔(G,E))≠*12-Int*$_s^\delta$(U,E)⊔*12-Int*$_s^\delta$(G,E).

Definition 4.4. Let ***(X,*$\tilde{\tau}_i$*,*$\tilde{\tau}_j$*, E)*** be a soft topological space and e_F∈***SP(X)*** is said to be a soft *ij-δ*-cluster point of (F,E)∈***SS(X,E)*** if for every a soft *ij*-regular open (U,E) soft containing of e_F we have (F,E)⊓(U,E)≠ $\mathbf{0_E}$. The set of all soft *ij-δ*-cluster points of (G,E) is called the soft *ij-δ*-closure denoted by *ij-Cl*$_s^\delta$(G,E).

Remark 4.2. Let (F,E) be a soft set of soft bitopological space ***(X,*$\tilde{\tau}_i$*,*$\tilde{\tau}_j$*, E)***. A soft *ij-δ*-closure of (G,E) in ***SS(X,E)*** is *ij-Cl*$_s^\delta$(F,E)=$\sqcap_{k\in I}${(V_k,E):(V_k,E)$\widetilde{\in}$ $\tilde{\tau}^c_{\delta ij}$,(F,E)⊑($V_k$,E)}.

Proposition 4.6. Let ***(X,*$\tilde{\tau}_i$*,*$\tilde{\tau}_j$*, E)*** be a soft bitopological space over X and (V,E), (F,E)∈***SS(X,E)***. Then.

(1) *ij-Cl*$_s^\delta$(V,E)∈***ij-SδC(X,E)***

(2) (V,E)⊑*ij-Cl*$_s^\delta$(V,E).

(3) (V,E) is a soft *ij-δ*-closed set if and only if *ij-Cl*$_s^\delta$(V,E)=(V,E).

(4) *ij-Cl*$_s^\delta$($\mathbf{0_E}$)= $\mathbf{0_E}$ and *ij-Cl*$_s^\delta$($\mathbf{1_E}$)=$\mathbf{1_E}$.

(5) *i-Cl*$_s$(V,E)⊑*ij-Cl*$_s^\delta$(V,E).where *i-Cl*$_s$(V,E) is the soft *i*-closure of (V,E) in ***(X,*$\tilde{\tau}_i$*,E)***

(6) *ij-Cl*$_s^\delta$(*ij-Cl*$_s^\delta$(F,E))=*ij-Cl*$_s^\delta$(F,E).

(7) (V,E)⊑(F,E) implies *ij-Cl*$_s^\delta$(V,E)⊑*ij-Cl*$_s^\delta$(F,E).

(8) *ij-Cl*$_s^\delta$(V,E)⊔*ij-Cl*$_s^\delta$(F,E)=*ij-Cl*$_s^\delta$((V,E)⊔(F,E)).

(9) *ij-Cl*$_s^\delta$((F,E)⊓(V,E))⊑*ij-Cl*$_s^\delta$(F,E)⊓*ij-Cl*$_s^\delta$(V,E).

proof. (1) and (2) Obvious.

(3) .Let (V,E)∈***ij-SδC(X,E)***, and There exists $e_F\widetilde{\in}$(U,E)∈***ij-SRO(X,E)***. If $e_F\widetilde{\in}$(V,E) then (V,E)⊓(U,E)≠ $\mathbf{0_E}$. Therefore $e_F\widetilde{\in}$*ij-Cl*$_s^\delta$(V,E). So (V,E)⊑*ij-Cl*$_s^\delta$(V,E). If $e_F\widetilde{\notin}$(V,E). Then (V,E)⊓(U,E)= $\mathbf{0_E}$. Therefore $e_F\widetilde{\notin}$*ij-Cl*$_s^\delta$(V,E). So *ij-Cl*$_s^\delta$(V,E)⊑(V,E).Thus if (V,E) be a soft *ij-δ*-closed set then *ij-Cl*$_s^\delta$(V,E)=(V,E).

Conversely, suppose that *ij*-Cl_s^{δ}(V,E)=(V,E). Then (V,E)∈***ij-SδC(X,E)***. Thus (V,E) is a soft *ij*-*δ*-closed set if and only if *ij*-Cl_s^{δ}(V,E) =(V,E).

(4), (5) and (6) are obvious.

(7) Pick $e_F$$\widetilde{\in}$*ij*-$Cl_s^{\delta}$(V,E) and (V,E)⊑(F,E). Then There exists a soft *ij*-regular open set (U,E) soft containing of e_F we have (V,E)⊓(U,E)≠ $\mathbf{0_E}$. Therefore (F,E)⊓(U,E)≠ $\mathbf{0_E}$. Then $e_F$$\widetilde{\in}$*ij*-$Cl_s^{\delta}$(V,E). Thus *ij*-$Cl_s^{\delta}$(V,E)⊑*ij*-$Cl_s^{\delta}$(F,E).

(8) Since (F,E)⊑(F,E)⊔(V,E) and(V,E)⊑(F,E)⊔(V,E). So by part (7),*ij*-Cl_s^{δ}(F,E)⊑*ij*-Cl_s^{δ}((F,E)⊔(V,E)) and *ij*-Cl_s^{δ}(V,E)⊑*ij*-Cl_s^{δ}((F,E)⊔(V,E)). Thus *ij*-Cl_s^{δ}(F,E)⊔*ij*-Cl_s^{δ}(V,E)⊑*ij*-Cl_s^{δ}((F,E)⊔(V,E)).

Conversely, suppose that (F,E)⊑*ij*-Cl_s^{δ}(F,E) and (V,E)⊑*ij*-Cl_s^{δ}(V,E). So (F,E)⊔(V,E)⊑*ij*-Cl_s^{δ}(F,E)⊔*ij*-Cl_s^{δ}(V,E). *ij*-Cl_s^{δ}(F,E)⊔*ij*-Cl_s^{δ}(V,E) is a soft *ij*-*δ*-closed set over X being the union of two soft δ-closed sets. Then *ij*-Cl_s^{δ}((F,E)⊔(V,E))⊑*ij*-Cl_s^{δ}(F,E)⊔*ij*-Cl_s^{δ}(V,E). Thus *ij*-Cl_s^{δ}((F,E)⊔(V,E))=*ij*-Cl_s^{δ}(F,E)⊔*ij*-Cl_s^{δ}(V,E).

(9) Since (F,E)⊓(V,E)⊑(F,E) and (F,E)⊓(V,E)⊑(V,E). So by part (8) *ij*-Cl_s^{δ}((F,E)⊓(V,E))⊑*ij*-Cl_s^{δ}(F,E) and *ij*-Cl_s^{δ}((F,E)⊓(V,E))⊑*ij*-Cl_s^{δ}(V,E). Thus *ij*-Cl_s^{δ}((F,E)⊓(V,E))⊑*ij*-Cl_s^{δ}(F,E)⊓*ij*-Cl_s^{δ}(V,E).

The following example shows that the equalities do not hold in Proposition 4.6(9)

Example 4.3. Refer Example 4.2. Suppose (H,E)=(U^c,E) and (M,E)=(G^c,E). We get *12*-Cl_s^{δ}((H,E)⊓(M,E))={(e_1,{h_1,h_4}),(e_1,{h_1,h_4})}, and *12*-Cl_s^{δ}(H,E)⊓*12*-Cl_s^{δ}(M,E)=$\mathbf{1_E}$. So *12*-Cl_s^{δ}((H,E)⊓(M,E))⊑*12*-Cl_s^{δ}(H,E)⊓*12*-Cl_s^{δ}(M,E). Therefore *12*-Cl_s^{δ}(H,E)⊓*12*-Cl_s^{δ}(M,E)≠*12*-Cl_s^{δ}((H,E)⊓(M,E)).

Proposition 4.7. Let***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)*** be a soft bitopological space, and (G,E)∈***SS(X,E)***. Then.

(1) (*ij*-Int_s^{δ}(G,E))^c=*ij*-Cl_s^{δ}(G^c,E).

(2) (*ij*-Cl_s^{δ}(G,E))^c=*ij*-Int_s^{δ}(G^c,E).

(3) *ij*-Int_s^{δ}(G,E)=(*ij*-Cl_s^{δ}(G^c,E))^c.

Proof. (1) (*ij*-Int_s^{δ}(G,E))^c=($\sqcup_{k\in I}${(U_k,E):(U_k,E)$\widetilde{\in}$ $\tilde{\tau}_{\delta ij}$,(U_k,E)⊑(G,E)^c}=

$\sqcap_{k\in I}${(U_k}^{c},E):(U_k^c,E)$\widetilde{\in}$ $\tilde{\tau}^c_{\delta ij}$,($U_k^c$,E)⊒($G^c$,E)}. Imply that $\sqcap_{k\in I}${(U_k^c,E):(U_k^c,E)∈$\tilde{\tau}^c_{\delta ij}$,($G^c$,E)⊑($U_k^c$,E)}=ij-$Cl_s^{\delta}$($G^c$,E)

(2) (*ij*-Cl_s^{δ}(G,E))^c=($\sqcap_{k\in I}${(F_k,E):(F_k,E)$\widetilde{\in}$ $\tilde{\tau}^c_{\delta ij}$,(G,E)⊑($F_k$,E)})^c

=$\sqcup_{k\in I}${(F_k^c,E):(F_k^c,E)$\widetilde{\in}$ $\tilde{\tau}_{\delta ij}$,(G^c,E)⊒(F_k^c,E)}, Thus $\sqcup_{k\in I}${(F_k^c,E):(F_k^c,E)$\widetilde{\in}$ $\tilde{\tau}_{\delta ij}$,(F_k^c,E)⊑(G^c,E)}=*ij*-Int_s^{δ}(G^c,E).

(3) Obvious

5. SOFT ij-δ-DERIVED SET, SOFT ij-δ-BORDER SET, SOFT ij-δ-FRONTIER SET AND SOFT ij-δ-EXTERIOR SET.

Definition 5.1. Let (G,E) be a soft set of a soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***. A soft point e_F∈***SP(X)*** is called soft*ij-δ*-limit point of (G,E) if for each soft *ij-δ*-open set (U,E) containing e_F, (U,E)⊓[(G,E)\{e_F})]≠$\mathbf{0_E}$. The set of all soft *ij-δ*-limit points of (G,E) is called the soft *ij-δ*-derived set of (G,E) and is denoted by $ij\text{-}D_s^{\delta}$(G,E).

Theorem 5.1. For soft subsets (G,E) and (U,E) of a soft bitopological space ***(X,$\tilde{\tau}_i$,$\tilde{\tau}_j$, E)***, the following are satisfied.

(1) $i\text{-}D_s$(G,E)⊑ $ij\text{-}D_s^{\delta}$(G,E) where $i\text{-}D_s$(G,E) is the derived set of (G,E) in ***(X,$\tilde{\tau}_i$,E)***.

(2) (G,E)⊑(U,E) implies $ij\text{-}D_s^{\delta}$(G,E)⊑$ij\text{-}D_s^{\delta}$(U,E).

(3) $ij\text{-}D_s^{\delta}$(G,E)⊔$ij\text{-}D_s^{\delta}$(U,E)=$ij\text{-}D_s^{\delta}$((G,E)⊔(U,E)) and $ij\text{-}D_s^{\delta}$((G,E)⊓(U,E))⊑$ij\text{-}D_s^{\delta}$(G,E)⊓$ij\text{-}D_s^{\delta}$(U,E).

(4) [$ij\text{-}D_s^{\delta}$($ij\text{-}D_s^{\delta}$(G,E))\(G,E)]⊑$ij\text{-}D_s^{\delta}$(G,E).

(5) $ij\text{-}D_s^{\delta}$((G,E)⊔$ij\text{-}D_s^{\delta}$(G,E))⊑(G,E)⊔$ij\text{-}D_s^{\delta}$(G,E).

Proof. (1) Obvious, since every *ij-δ*-open set is open.

(2) Obvious.

(3) Since (G,E)⊑(G,E)⊔(U,E) and (U,E)⊑(G,E)⊔(U,E), then $ij\text{-}D_s^{\delta}$(G,E)⊑$ij\text{-}D_s^{\delta}$((G,E)⊔(U,E)) and$ij\text{-}D_s^{\delta}$(U,E)⊑$ij\text{-}D_s^{\delta}$((G,E)⊔(U,E)). Therefore $ij\text{-}D_s^{\delta}$(G,E)⊔$ij\text{-}D_s^{\delta}$(U,E)⊑$ij\text{-}D_s^{\delta}$((G,E)⊔(U,E)). Now, let $e_F$$\tilde{\notin}$$ij\text{-}D_s^{\delta}$(G,E)⊔$ij\text{-}D_s^{\delta}$(U,E). Then $e_F$$\tilde{\notin}$$ij\text{-}D_s^{\delta}$(G,E) and $e_F$$\tilde{\notin}$$ij\text{-}D_s^{\delta}$(U,E). Then there exist an *ij-δ*-open set (U,E) containing e_F and an *ij-δ*-open set (V,E) containing e_F such that (U,E)⊓((G,E)\{e_F})=$\mathbf{0_E}$and (V,E)⊓((U,E)\{e_F})=$\mathbf{0_E}$. Then (U,E)⊓(V,E)⊓(((G,E)⊔(U,E))\{e_F})=$\mathbf{0_E}$. Then $e_F$$\tilde{\notin}$$ij\text{-}D_s^{\delta}$((G,E)⊔(U,E)) and therefore $ij\text{-}D_s^{\delta}$((G,E)⊔(U,E))⊑$ij\text{-}D_s^{\delta}$(G,E)⊔$ij\text{-}D_s^{\delta}$(U,E). By (2) $ij\text{-}D_s^{\delta}$((G,E)⊓(U,E))⊑$ij\text{-}D_s^{\delta}$(G,E) and $ij\text{-}D_s^{\delta}$((G,E)⊓(U,E))⊑$ij\text{-}D_s^{\delta}$(U,E). Therefore, $ij\text{-}D_s^{\delta}$((G,E)⊓(U,E))⊑$ij\text{-}D_s^{\delta}$(G,E)⊓$ij\text{-}D_s^{\delta}$(U,E).

(4) Let $e_F$$\tilde{\in}$[$ij\text{-}D_s^{\delta}$($ij\text{-}D_s^{\delta}$(G,E))\(G,E)] and (U,E) be an *ij-δ*-open set containing e_F. Then (U,E)⊓($ij\text{-}D_s^{\delta}$(G,E)\{e_F})≠$\mathbf{0_E}$. Let $e_V$$\tilde{\in}$(U,E)⊓($ij\text{-}D_s^{\delta}$(G,E)\{$e_F$}). Then, since $e_v$$\tilde{\in}$$ij\text{-}D_s^{\delta}$(G,E) and $e_v$$\tilde{\in}$(U,E), (U,E)⊓[(G,E)\{$e_v$]≠$\mathbf{0_E}$ Let $e_H$$\tilde{\in}$(U,E)⊓[(G,E)\{$e_V$}}]. Then e_H≠ e_F for $e_H$$\tilde{\in}$(G,E) and $e_F$$\tilde{\notin}$(G,E). Hence (U,E)⊓[(G,E)\{e_F}]≠$\mathbf{0_E}$. Therefore $e_F$$\tilde{\in}$$ij\text{-}D_s^{\delta}$(G,E).

(5) Let $e_F$$\tilde{\in}$$ij\text{-}D_s^{\delta}$((G,E)⊔$ij\text{-}D_s^{\delta}$(G,E)). If $e_F$$\tilde{\in}$(G,E), the result is obvious. So, let $e_F$$\tilde{\in}$[$ij\text{-}D_s^{\delta}$((G,E)⊔$ij\text{-}D_s^{\delta}$(G,E))\(G,E)] . Then for an *ij-δ*-open set (U,E) containing e_F, (U,E)⊓[(G,E)⊔$ij\text{-}D_s^{\delta}$(G,E)\{e_F}]≠$\mathbf{0_E}$. Thus (U,E)⊓((G,E)\{e_F})≠$\mathbf{0_E}$. or (U,E)⊓($ij\text{-}D_s^{\delta}$(G,E)\{e_F})≠$\mathbf{0_E}$. It follows from (4) that (U,E)⊓((G,E)\{e_F})≠$\mathbf{0_E}$. Hence $e_F$$\tilde{\in}$$ij\text{-}D_s^{\delta}$(G,E). Therefore, in any case, $ij\text{-}D_s^{\delta}$((G,E)⊔$ij\text{-}D_s^{\delta}$(G,E))⊑(G,E)⊔$ij\text{-}D_s^{\delta}$(G,E). □

In general, the reverse inclusions in (1) and (3) above may not be true as shown by the following examples:

Example 5.1. The soft bitopological space (X,τ_1,τ_2,E) is the same as in Example 4.1 Suppose that $(G,E)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_1,h_3,h_4\})\}$. We can see that $12\text{-}D_s^{\delta}(G,E)=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$, $1\text{-}D_s(G,E)=\{(e_1,\{h_4\}),(e_2,\{h_4\})\}$. Then $1\text{-}D_s(G,E)\sqsubseteq 12\text{-}D_s^{\delta}(G,E)$, But the reverse is not true.

Example 5.2. The soft bitopological space (X,τ_1,τ_2,E) is the same as in Example 4.1 Suppose that and let $(G,E)=\{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$, $(U,E)=\{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$. Then $(G,E)\sqcap(U,E)=\{(e_1,\{h_3\}),(e_2,\{h_3\})\}$. Now $12\text{-}D_s^{\delta}(G,E)=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$, $12\text{-}D_s^{\delta}(U,E)=\{(e_1,\{h_1,h_3,h_4\}),(e_2,\{h_1,h_3,h_4\})\}$,and $12\text{-}D_s^{\delta}((G,E)\sqcap(U,E))=\mathbf{0_E}$, $12\text{-}D_s^{\delta}(G,E)\sqcap 12\text{-}D_s^{\delta}(U,E)=\{(e_1,\{h_3,h_4\}),(e_2,\{,h_3,h_4\})\}$. Then $12\text{-}D_s^{\delta}((G,E)\sqcap(U,E))\sqsubseteq 12\text{-}D_s^{\delta}(G,E)\sqcap 12\text{--}D_s^{\delta}(U,E)$. But the reverse is not true.

Theorem 5.2. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$be a soft bitopological space and $(G,E)\in SS(X,E)$,$ij\text{-}Cl_s^{\delta}(G,E)=(G,E)\sqcup ij\text{-}D_s^{\delta}(G,E)$.

Proof. Since $ij\text{-}D_s^{\delta}(G,E)\sqsubseteq ij\text{-}Cl_s^{\delta}(G,E)$,$(G,E)\sqcup ij\text{-}D_s^{\delta}(G,E)\sqsubseteq ij\text{-}Cl_s^{\delta}(G,E)$. On the other hand let $e_F\tilde{\in} ij\text{-}Cl_s^{\delta}(G,E)$. If $e_F\tilde{\in}(G,E)$, then the proof is complete. If $e_F\widetilde{\notin}(G,E)$, each ij-δ-open set (U,E) containing e_F intersects (G,E) at a soft point distinct from e_F, so $e_F\tilde{\in} ij\text{-}D_s^{\delta}(G,E)$. Thus $ij\text{-}Cl_s^{\delta}(G,E)\sqsubseteq(G,E)\sqcup ij\text{-}D_s^{\delta}(G,E)$, which completes the proof.

Corollary 5.1. A soft set (G,E) of a soft bitopological space $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$is ij-δ-closed if and only if it contains all of its ij-δ-limit points.

Definition 5.2. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$ be a soft bitopological space and (U,E) be a soft set over X. The soft ij-δ-border of soft set (U,E) over X is denoted by $ij\text{-}B_s^{\delta}(U,E)$, is defined by $ij\text{-}B_s^{\delta}(U,E)=(U,E)\setminus ij\text{-}Int_s^{\delta}(U,E)$.

Theorem 5.3. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$ be a soft topological space, $(U,E)\in SS(X,E)$. Then the following statements are true.

(1) $i\text{-}B_s(U,E)\sqsubseteq ij\text{-}B_s^{\delta}(U,E)$, where $i\text{-}B_S(U,E)=(U,E)\setminus i\text{-}Int_s(U,E)$.

(2) $(U,E)=ij\text{-}Int_s^{\delta}(U,E)\sqcup ij\text{-}B_s^{\delta}(U,E)$.

(3) $ij\text{-}Int_s^{\delta}(U,E)\sqcap ij\text{-}B_s^{\delta}(U,E)=\mathbf{0_E}$.

(4) (U,E) is an ij-δ-open set if and only if $ij\text{-}B_s^{\delta}(U,E)=\mathbf{0_E}$.

(5) $ij\text{-}B_s^{\delta}(ij\text{-}Int_s^{\delta}(U,E))=\mathbf{0_E}$.

(6) $ij\text{-}Int_s^{\delta}(ij\text{-}B_s^{\delta}(U,E))=\mathbf{0_E}$.

(7) $ij\text{-}B_s^{\delta}(ij\text{-}B_s^{\delta}(U,E))=ij\text{-}B_s^{\delta}(U,E)$.

(8) $ij\text{-}B_s^{\delta}(U,E)=(U,E)\sqcap ij\text{-}Cl_s^{\delta}((\mathbf{1_E})\setminus(U,E))$.

(9) $ij\text{-}B_s^{\delta}(U,E)=ij\text{-}D_s^{\delta}((\mathbf{1_E})\setminus(U,E))$.

Proof. (1) Since $ij\text{-}Int_s^{\delta}(U,E)\sqsubseteq i\text{-}Int_s(U,E)$, Then $i\text{-}B_S(U,E)=(U,E)\setminus i\text{-}Int_s(U,E)\sqsubseteq(U,E)\setminus ij\text{-}Int_s^{\delta}(U,E)=ij\text{-}B_s^{\delta}(U,E)$.

(2) and (3). Straightforward.

(4) Let (U,E) is soft ij-δ-open. This means that $(U,E)=ij\text{-}Int_s^{\delta}(U,E)$, Leads to that$ij\text{-}B_s^{\delta}(U,E)=(U,E)\setminus ij\text{-}Int_s^{\delta}(U,E)=\mathbf{0_E}$.

Conversely, obvious

(5) Since $ij\text{-}Int_s^{\delta}(U,E)$ is soft ij-δ-open, it follows from (4) that $ij\text{-}B_s^{\delta}(ij\text{-}Int_s^{\delta}(U,E))=\mathbf{0_E}$.

(6) Let $e_F\tilde{\in} ij\text{-}Int_s^{\delta}(ij\text{-}B_s^{\delta}(U,E))$. Then $e_F\tilde{\in} ij\text{-}B_s^{\delta}(U,E)$. On the other hand since $ij\text{-}B_s^{\delta}(U,E) \sqsubseteq (U,E)$, $e_F\tilde{\in} ij\text{-}Int_s^{\delta}(ij\text{-}B_s^{\delta}(U,E))\sqsubseteq ij\text{-}Int_s^{\delta}(U,E)$. Hence $e_F\tilde{\in} ij\text{-}B_s^{\delta}(U,E)\sqcap ij\text{-}Int_s^{\delta}(U,E)$ which contradicts (3). Therefore $ij\text{-}Int_s^{\delta}(ij\text{-}B_s^{\delta}(U,E))=\mathbf{0_E}$.

(8) $ij\text{-}B_s^{\delta}(U,E)=(U,E)\setminus ij\text{-}Int_s^{\delta}(U,E)=(U,E)\setminus((\mathbf{1_E})\setminus ij\text{-}Cl_s^{\delta}((\mathbf{1_E})\setminus(U,E)))=(U,E)\sqcap(ij\text{-}Cl_s^{\delta}((\mathbf{1_E})\setminus(U,E)))$.

(9) $ij\text{-}B_s^{\delta}(U,E)=(U,E)\setminus ij\text{-}Int_s^{\delta}(U,E)=(U,E)\setminus((U,E)\setminus ij\text{-}D_s^{\delta}((\mathbf{1_E})\setminus(U,E)))=ij\text{-}D_s^{\delta}((\mathbf{1_E})\setminus(U,E))$.

Definition 5.3. Let (G,E) be a soft set of soft bitopological space $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$, the soft ij-δ-frontier of (G,E), denoted by $ij\text{-}Fr_S^{\delta}(G,E)$ is defined by

$ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)$.

Theorem 5.4. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$ be a soft bitopological space and (G,E)∈***SS(X,E)***, the following are satisfied:

(1) $i\text{-}Fr_s(G,E) \sqsubseteq ij\text{-}Fr_S^{\delta}(G,E)$ where $i\text{-}Fr_s(G,E)=i\text{-}Cl_s(G,E)\setminus i\text{-}Int_s(G,E)$.

(2) $ij\text{-}Cl_s^{\delta}(G,E)=ij\text{-}Int_s^{\delta}(G,E)\sqcup ij\text{-}Fr_S^{\delta}(G,E)$.

(3) $ij\text{-}Int_s^{\delta}(G,E)\sqcap ij\text{-}Fr_S^{\delta}(G,E)=\mathbf{0_E}$.

(4) $ij\text{-}B_s^{\delta}(G,E)\sqsubseteq ij\text{-}Fr_S^{\delta}(G,E)$.

(5) $ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}B_s^{\delta}(G,E)\sqcup ij\text{-}D_s^{\delta}(G,E)$.

(6) (G,E) is ij-δ-open if and only if $ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}D_s^{\delta}(G,E)$.

(7) $ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)\sqcap ij\text{-}Cl_s^{\delta}(\mathbf{1_E}\setminus(G,E))$.

(8) $ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Fr_S^{\delta}(\mathbf{1_E}\setminus(G,E))$.

(9) $ij\text{-}Fr_S^{\delta}(G,E)$ is ij-δ-closed.

(10) $ij\text{-}Fr_S^{\delta}(ij\text{-}Fr_S^{\delta}(G,E))\sqsubseteq ij\text{-}Fr_S^{\delta}(G,E)$.

Proof. (1) $e_F\tilde{\in} i\text{-}Fr_s(G,E) \Rightarrow e_F\tilde{\in} i\text{-}Cl_s(G,E)$ and $e_F\tilde{\notin} i\text{-}Int_s(G,E) \Rightarrow e_F\tilde{\in} ij\text{-}Cl_s^{\delta}(G,E)$ and $e_F\tilde{\notin} ij\text{-}Int_s^{\delta}(G,E) \Rightarrow e_F\tilde{\in} ij\text{-}Cl_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}Fr_S^{\delta}(G,E)$.

(2) $ij\text{-}Int_s^{\delta}(G,E)\sqcup ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Int_s^{\delta}(G,E)\sqcup(ij\text{-}Cl_s^{\delta}(G,E)\text{-}ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)$.

(3) $ij\text{-}Int_s^{\delta}(G,E)\sqcap ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Int_s^{\delta}(G,E)\sqcap(ij\text{-}Cl_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E))=\mathbf{0_E}$.

(4) $ij\text{-}Bd_s^{\delta}(G,E)=(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)\sqsubseteq ij\text{-}Cl_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}Fr_S^{\delta}(G,E)$.

(5) $ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)=(G,E)\sqcup ij\text{-}D_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}D_s^{\delta}(G,E)\sqcup(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}D_s^{\delta}(G,E)\sqcup ij\text{-}B_s^{\delta}(G,E)$.

(6) $ij\text{-}Fr_S^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)\setminus ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)\sqcap(\mathbf{1_E}\setminus ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}Cl_s^{\delta}(G,E)\sqcap ij\text{-}Cl_s^{\delta}(\mathbf{1_E}\setminus(G,E))$.

(7) $ij\text{-}Cl_s^{\delta}(ij\text{-}Fr_S^{\delta}(G,E))=ij\text{-}Cl_s^{\delta}(ij\text{-}Cl_s^{\delta}(G,E)\sqcap ij\text{-}Cl_s^{\delta}(\mathbf{1_E}\backslash(G,E))\sqsubseteq ij\text{-}Cl_s^{\delta}(ij\text{-}Cl_s^{\delta}(G,E))\sqcap ij\text{-}Cl_s^{\delta}(ij\text{-}Cl_s^{\delta}(\mathbf{1_E}\backslash(G,E))=ij\text{-}Cl_s^{\delta}(G,E)\sqcap ij\text{-}Cl_s^{\delta}(\mathbf{1_E}\backslash(G,E))=ij\text{-}Fr_S^{\delta}(G,E)$. Therefore $ij\text{-}Fr_S^{\delta}(G,E)$ is*ij-δ*-closed.

(8) $ij\text{-}Fr_S^{\delta}(ij\text{-}Fr_S^{\delta}(G,E))=ij\text{-}Cl_s^{\delta}(ij\text{-}Fr_S^{\delta}(G,E))\sqcap ij\text{-}Cl_s^{\delta}(\mathbf{1_E}\backslash ij\text{-}Fr_S^{\delta}(G,E))\sqsubseteq ij\text{-}Cl_s^{\delta}(ij\text{-}Fr_S^{\delta}(G,E))=ij\text{-}Fr_S^{\delta}(G,E)$.

(9) From (7)

(10) $ij\text{-}Fr_S^{\delta}(ij\text{-}Cl_s^{\delta}(G,E))=ij\text{-}Cl_s^{\delta}(ij\text{-}Cl_s^{\delta}(G,E)\backslash ij\text{-}Int_s^{\delta}(ij\text{-}Cl_s^{\delta}(G,E))=ij\text{-}Cl_s^{\delta}(G,E)\backslash ij\text{-}Int_s^{\delta}(ij\text{-}Cl_s^{\delta}(G,E))\sqsubseteq ij\text{-}Cl_s^{\delta}(G,E)\backslash ij\text{-}Int_s^{\delta}(G,E)=ij\text{-}Fr_S^{\delta}(G,E)$..

The converses of Theorem 5.4(1 and 4) above are not true, in general, as shown by the following examples:

Example 5.3. The soft bitopological space ***(X,τ₁,τ₂,E)*** is the same as in Example 4.1 and $(G,E)=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$. Then $1\text{-}Cl_s(G,E)=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$, $1\text{-}Int_s(G,E)=\{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$. Then $1\text{-}Fr_s(G,E)=\{(e_1,\{,h_4\}),(e_2,\{h_4\})\}$. Now, $12\text{-}Cl_s^{\delta}(G,E)=\mathbf{1_E}$, $12\text{-}Int_s^{\delta}(G,E)=\mathbf{0_E}$. Then $12\text{-}Fr_S^{\delta}(G,E)=\mathbf{1_E}$. Then $1\text{-}Fr_s(G,E)\sqsubseteq 12\text{-}Fr_S^{\delta}(G,E)$.But the reverse is not true.

Example 5.4. By Example 5.3. Now

$12\text{-}B_s^{\delta}(G,E)=(G,E)\backslash 12\text{-}Int_s^{\delta}(G,E)=(G,E)\backslash\mathbf{0_E}=\{(e_1,\{h_2,h_3,h_4\}),(e_2,\{h_2,h_3,h_4\})\}$. Then $12\text{-}B_S^{\delta}(G,E)\sqsubseteq 12\text{-}Fr_S^{\delta}(G,E)$.But the reverse is not true.

Remark 5.2. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$be a soft bitopological space and (G,E), (U,E)∈***SS(X,E)***.Then (G,E)⊑(U,E) does not imply that either $ij\text{-}Fr_S^{\delta}(G,E)\sqsubseteq ij\text{-}Fr_S^{\delta}(U,E)$ or $ij\text{-}Fr_S^{\delta}(U,E)\sqsubseteq ij\text{-}Fr_S^{\delta}(G,E)$.

Definition 5.4. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$ be a soft bitopological space and (F,E)∈***SS(X,E)***. The *ij-δ*-exterior of (F,E) is the soft set. $ij\text{-}Ext_S^{\delta}(F,E)=ij\text{-}Int_s^{\delta}(F^c,E)$ is said to be a soft *ij-δ*-exterior of (F,E).

Theorem 5.5. Let $(X,\tilde{\tau}_i,\tilde{\tau}_j, E)$ be a soft bitopological space and (F,E), (G,E)∈***SS(X,E)***. Then the following statements hold.

(1) $ij\text{-}Ext_S^{\delta}(F,E)\sqsubseteq i\text{-}Ext_s(F,E)$ where $i\text{-}Ext_s(F,E)=i\text{-}Int_s(F^c,E)$.

(2) $ij\text{-}Ext_S^{\delta}(F,E)$ is soft *ij-δ*-open set.

(3) $ij\text{-}Ext_S^{\delta}(F,E)=\mathbf{1_E}\backslash ij\text{-}Cl_s^{\delta}(F,E)$.

(4) $ij\text{-}Ext_S^{\delta}(ij\text{-}Ext_S^{\delta}(F,E))=ij\text{-}Int_s^{\delta}(ij\text{-}Cl_s^{\delta}(F,E))$.

(5) If (F,E)⊑(G,E), then $ij\text{-}Ext_S^{\delta}(F,E)\sqsupseteq ij\text{-}Ext_S^{\delta}(G,E)$.

(6) $ij\text{-}Ext_S^{\delta}((F,E)\sqcup(G,E))=ij\text{-}Ext_S^{\delta}(F,E)\sqcap ij\text{-}Ext_S^{\delta}(G,E)$.

(7) $ij\text{-}Ext_S^{\delta}(F,E)\sqcap ij\text{-}Ext_S^{\delta}(G,E)\sqsubseteq ij\text{-}Ext_S^{\delta}((F,E)\sqcap(G,E))$.

(8) $ij\text{-}Ext_S^{\delta}(\mathbf{1_E})=\mathbf{0_E}$, and $ij\text{-}Ext_S^{\delta}(\mathbf{0_E})=\mathbf{1_E}$.

(9) $ij\text{-}Ext_S^{\delta}((ij\text{-}Ext_S^{\delta}(F,E))$^{c})=$ij\text{-}Ext_S^{\delta}(F,E)$.

(10) $ij\text{-}Ext_s^{\delta}$(F,E)⊔$ij\text{-}Ext_s^{\delta}$(G,E)⊑$ij\text{-}Ext_s^{\delta}$((F,E)⊓(G,E)).

(11) $ij\text{-}Int_s^{\delta}$(F,E)⊑$ij\text{-}Ext_s^{\delta}$($ij\text{-}Ext_s^{\delta}$(F,E)).

(12) $ij\text{-}Fr_s^{\delta}$(F,E)⊔$ij\text{-}Int_s^{\delta}$(F,E)⊔$ij\text{-}Ext_s^{\delta}$(F,E)=$\mathbf{1_E}$

Proof. (1) $ij\text{-}Ext_s^{\delta}$(F,E)=$ij\text{-}Int_s^{\delta}$(F^c,E)⊑$i\text{-}Int_s$(F^c,E)=$i\text{-}Ext_s$(F,E)

(2) Since $ij\text{-}Int_s^{\delta}$(F^c,E) is soft ij-δ-open set Then $ij\text{-}Ext_s^{\delta}$(F,E) is soft ij-δ-open set

(3) Obvious

(4) $ij\text{-}Ext_s^{\delta}$($ij\text{-}Ext_s^{\delta}$(F,E))=$ij\text{-}Int_s^{\delta}$($ij\text{-}Ext_s^{\delta}$(F^c,E))=$ij\text{-}Int_s^{\delta}$($ij\text{-}Int_s^{\delta}$(F^c,E))^c=$ij\text{-}Int_s^{\delta}$(($ij\text{-}Cl_s^{\delta}$(F,E))^c)^c=$ij\text{-}Int_s^{\delta}$($ij\text{-}Cl_s^{\delta}$(F,E)).

(5) If (F,E)⊑(G,E)then (F^c,E)⊒(G^c,E) and $ij\text{-}Int_s^{\delta}$(F^c,E)⊒$ij\text{-}Int_s^{\delta}$(G^c,E). Thus $ij\text{-}Ext_s^{\delta}$(F,E)⊒$ij\text{-}Ext_s^{\delta}$(G,E)

(6) $ij\text{-}Ext_s^{\delta}$((F,E)⊔(G,E))=$ij\text{-}Int_s^{\delta}$(((F,E)⊔(G,E))^c)=$ij\text{-}Int_s^{\delta}$((F^c,E)⊓(G^c,E))=$ij\text{-}Int_s^{\delta}$(F^c,E)⊓$ij\text{-}Int_s^{\delta}$(G^c,E)=$ij\text{-}Ext_s^{\delta}$(F,E)⊓$ij\text{-}Ext_s^{\delta}$(G,E)

(7) $ij\text{-}Ext_s^{\delta}$(F,E)⊓$ij\text{-}Ext_s^{\delta}$(G,E)=$ij\text{-}Int_s^{\delta}$(F^c,E)⊓$ij\text{-}Int_s^{\delta}$(G^c,E)⊑$ij\text{-}Int_s^{\delta}$((F^c,E)⊓(G^c,E))=$ij\text{-}Int_s^{\delta}$((F,E)⊔(G,E))^c=$ij\text{-}Ext_s^{\delta}$((F,E)⊔(G,E))⊑$ij\text{-}Ext_s^{\delta}$((F,E)⊓(G,E))

(8) $ij\text{-}Ext_s^{\delta}$($\mathbf{1_E}$)=$ij\text{-}Int_s^{\delta}$($\mathbf{1_E}$)^c=$ij\text{-}Int_s^{\delta}$($\mathbf{0_E}$)=$\mathbf{0_E}$ and $ij\text{-}Ext_s^{\delta}$($\mathbf{0_E}$)=$ij\text{-}Int_s^{\delta}$($\mathbf{0_E}$)^c=$ij\text{-}Int_s^{\delta}$($\mathbf{1_E}$)=$\mathbf{1_E}$

(9) $ij\text{-}Ext_s^{\delta}$($ij\text{-}Ext_s^{\delta}$(F,E))^c=$ij\text{-}Ext_s^{\delta}$($ij\text{-}Int_s^{\delta}$(F^c,E))^c=$ij\text{-}Int_s^{\delta}$(($ij\text{-}Int_s^{\delta}$(F^c,E))^c)^c=$ij\text{-}Int_s^{\delta}$($ij\text{-}Int_s^{\delta}$(F^c,E))=$ij\text{-}Int_s^{\delta}$(F^c,E)=$ij\text{-}Ext_s^{\delta}$(F,E).

(10) $ij\text{-}Ext_s^{\delta}$(F,E)⊔$ij\text{-}Ext_s^{\delta}$(G,E)=$ij\text{-}Int_s^{\delta}$(F^c,E)⊔$ij\text{-}Int_s^{\delta}$(G^c,E)⊑$ij\text{-}Int_s^{\delta}$((F^c,E)⊔(G^c,E))=$ij\text{-}Int_s^{\delta}$(((F,E)⊓(G,E))^c) =$ij\text{-}Ext_s^{\delta}$((F,E)⊓(G,E)).

(11) $ij\text{-}Int_s^{\delta}$(F,E)⊑$ij\text{-}Int_s^{\delta}$($ij\text{-}Cl_s^{\delta}$(F,E))=$ij\text{-}Int_s^{\delta}$($ij\text{-}Int_s^{\delta}$(F^c,E))^c=$ij\text{-}Int_s^{\delta}$($ij\text{-}Ext_s^{\delta}$(F,E))^c=$ij\text{-}Ext_s^{\delta}$($ij\text{-}Ext_s^{\delta}$(F,E)).

(12) $ij\text{-}Fr_s^{\delta}$(F,E)⊔$ij\text{-}Int_s^{\delta}$(F,E)⊔$ij\text{-}Ext_s^{\delta}$(F,E)=$ij\text{-}Cl_s^{\delta}$(F,E)⊔($\mathbf{1_E}$\$ij\text{-}Cl_s^{\delta}$(F,E))=$\mathbf{1_E}$

□

The following example shows that the equalities do not hold in Theorem 5.8 (7),(10) and (11)

Example 5.5. By Example 4.2 (7), we heve $12\text{-}Int_s^{\delta}$(U^c,E)⊓$12\text{-}Int_s^{\delta}$(G^c,E)={(e_1,{h_1,h_2,h_4}),(e_1,{h_1,h_2,h_4})} and $12\text{-}Int_s^{\delta}$((U^c,E)⊓(G^c,E))=$\mathbf{1_E}$. We obtain $12\text{-}Ext_s^{\delta}$(U,E)⊓$12\text{-}Ext_s^{\delta}$(G,E)⊑$12\text{-}Ext_s^{\delta}$((U,E)⊓(G,E)), and $12\text{-}Ext_s^{\delta}$((U,E)⊓(G,E))≠$12\text{-}Ext_s^{\delta}$(U,E)⊓$12\text{-}Ext_s^{\delta}$(G,E). Similarly, we find that (10) $12\text{-}Int_s^{\delta}$(U^c,E)⊔$12\text{-}Int_s^{\delta}$(G^c,E)=$\mathbf{0_E}$. Than $12\text{-}Ext_s^{\delta}$(U,E)⊔$12\text{-}Ext_s^{\delta}$(G,E)⊑$12\text{-}Ext_s^{\delta}$((U,E)⊓(G,E)), but $12\text{-}Ext_s^{\delta}$(U,E)⊔$12\text{-}Ext_s^{\delta}$(G,E)≠$12j\text{-}Ext_s^{\delta}$((U,E)⊓(G,E)). In Example 4.2 (11), we have $12\text{-}Int_s^{\delta}$(U,E)=$\mathbf{0_E}$, and $12\text{-}Ext_s^{\delta}$($12\text{-}Ext_s^{\delta}$(U,E))=$12\text{-}Ext_s^{\delta}$($12\text{-}Int_s^{\delta}$(U^c,E))=$12\text{-}Ext_s^{\delta}$($\mathbf{0_E}$)=$\mathbf{1_E}$. Than $12\text{-}Int_s^{\delta}$(U,E)⊑$12\text{-}Ext_s^{\delta}$($12\text{-}Ext_s^{\delta}$(U,E)).

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