

Application of Order-Statistic Filtering Based on K-Value in Image Denoising

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Abstract: Image denoising serves as a critical preprocessing step in digital image processing. In this study, we evaluate the performance of order-statistic filtering with varying k-values for salt-and-pepper noise removal, using a 3×3 sliding window and adopting mean filtering as a benchmark. Quantitative assessment is carried out using PSNR, SNR, and SSIM as evaluation metrics. Our experimental results demonstrate that order-statistic filtering with k=5, i.e., standard median filtering, achieves the optimal denoising performance. Smaller values of k tend to produce darker outputs, while larger values yield brighter reconstructions. In contrast, mean filtering oversmooths fine details and yields substantially inferior performance compared to order-statistic filtering across all metrics considered. Overall, these findings highlight the critical role of k-value selection and offer both theoretical insights and experimental validation for practical image denoising applications.

Keywords: *Order-statistic filtering, k-value selection, median filtering, max filtering, min filtering*

I. Introduction

Digital images are highly susceptible to salt-and-pepper noise during acquisition and transmission. Such contamination significantly impairs visual quality and impedes subsequent analytical processes. Consequently, image denoising persists as a perennially active research domain within image processing, with pervasive applications spanning remote sensing, target tracking, and biomedical image analysis [4-5]. Conventional mean filtering tends to excessively smooth fine details; in contrast, while standard median filtering effectively suppresses salt-and-pepper noise, it frequently compromises edge integrity and textural information, particularly under conditions of elevated noise density.

Alternative denoising approaches, such as MV-algebra-based methods and non-local frameworks

specifically designed for ultrasound imaging, have markedly improved noise suppression in clinical practice. As imaging technologies continue to evolve, the requirements imposed by medical applications become increasingly rigorous, particularly in terms of detail preservation and real-time processing capability. Techniques including multi-atlas segmentation, sparse representation fusion, and adaptive fuzzy filtering have further expanded the theoretical foundation of image denoising [6-9]. In the present study, we adopt a fixed 3×3 filtering window and utilize mean filtering as a baseline to systematically examine the influence of varying k-values on the denoising efficacy of order-statistic filtering. Quantitative assessment is conducted using PSNR, SNR, and SSIM as evaluation metrics. Based on comprehensive experimental analyses, we determine the optimal

k-value and offer practical recommendations for salt-and-pepper noise removal.

II. Related Work and Basic Theory

2.1 Overview of Classical Filtering Methods

(1) Median filtering, which substitutes the central pixel with the median value of its local neighborhood, achieves robust suppression of salt-and-pepper noise while effectively preserving edge information [1].

$$g(i, j) = \text{Med}_{(x,y) \in S} \{f(x, y)\} \quad (1)$$

where $g(i,j)$ denotes the output pixel value, $f(x,y)$ represents the input pixel value within the window S , and Med is the median operator.

(2) Max filtering, which substitutes the central pixel with the maximum value within its local neighborhood, effectively eliminates black-spot noise but tends to elevate overall image brightness [2].

$$g(i, j) = \text{Max}_{(x,y) \in S} \{f(x, y)\} \quad (2)$$

where $g(i,j)$ denotes the filtered pixel value at coordinates (i,j) , $f(x,y)$ is the input pixel value at (x,y) within the neighborhood window S , Max denotes the maximum operator, and S represents the set of pixels in that neighborhood window.

(3) Min filtering, which replaces the central pixel with the minimum value within its local neighborhood, effectively suppresses white-spot noise but tends to darken the overall image [2].

$$g(i, j) = \text{Min}_{(x,y) \in S} \{f(x, y)\} \quad (3)$$

where $g(i,j)$ denotes the filtered pixel value at coordinates (i,j) , $f(x,y)$ is the input pixel value at (x,y) within the neighborhood window S , Min denotes the minimum operator, and S represents the set of pixels comprising that window.

2.2 Principle of Order-Statistic Filtering

Order-statistic filtering is a nonlinear spatial filtering technique that operates on the basis of local region ranking. Its fundamental procedure can be described as follows: a sliding window of fixed size (specifically, a 3×3 window in this study) is defined; the filter iterates over every pixel of the input image, extracting all gray-level values within the window and sorting them in ascending order; the k -th value in the sorted sequence is then selected as the output for the central pixel. Upon completing the traversal

of the entire image, the final filtered result is produced [3].

$$g(i, j) = \text{Sort}_k \{f(x, y) | (x, y) \in S\} \quad (4)$$

where Sort_k denotes the operator that selects the k -th smallest value from the sorted sequence within the window S .

2.3 Physical Meaning of k-value

Within a 3×3 window, the nine pixel values are sorted and assigned ranks from 1 to 9. The parameter k directly governs the filtering behavior: $k=1$ corresponds to minimum filtering; $k=5$ to median filtering; and $k=9$ to maximum filtering, while intermediate values of k (i.e., $k=2,3,4$) yield transitional characteristics between the minimum and median filters.

2.4 Degradation Characteristics of Order-Statistic Filtering

Order-statistic filtering constitutes a unified framework. When the parameter k takes specific values, it naturally reduces to classical filter types: $k=1$ corresponds to minimum filtering, $k=5$ to median filtering, and $k=9$ to maximum filtering. This observation underscores the generality and representativeness of the order-statistic approach.

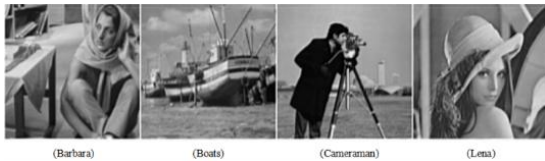
III. Experimental Design and Implementation

3.1 Dataset Description

This study employs the USC-SIPI standard test image set commonly used in the field of image processing, and selects four classic images including Lena, Barbara, Boats and Cameraman as experimental samples. The images cover various scenes such as portraits, still lifes, and landscapes, with texture complexity ranging from low to high. They contain rich edges, skin tones, hair, periodic stripes, and large smooth areas, which can thoroughly evaluate the salt-and-pepper noise removal effect and detail preservation ability of the order-statistic filtering algorithm in different scenes. All images are converted to 8-bit grayscale format with uniform standard resolution. Salt-and-pepper noise with a density of 0.05 is artificially added to construct experimental samples, providing a

universal, reproducible and comparable benchmark environment for algorithm performance evaluation.

Fig. 1 Original grayscale images



3.2 Evaluation Metrics

Signal-to-Noise Ratio (SNR) is essentially the ratio of the signal power spectrum to the noise power spectrum. Since the power spectrum is difficult to obtain directly; therefore, the ratio of image signal variance to noise variance is commonly used as an approximation in engineering. The specific method is: count the local variance of all pixels of the image, take the maximum local variance as the signal variance and the minimum as the noise variance. The higher the SNR value, the less noise the image contains and the better the imaging quality. The mathematical definition of SNR is as follows:

$$SNR = 10 \lg \left(\frac{P_s}{P_n} \right) \quad (5)$$

where P_s represents the effective power of the signal and P_n represents the effective power of the noise. Peak Signal-to-Noise Ratio (PSNR) is solved based on the pixel error between the original image and the processed image, which is a classic index for evaluating the degree of image coding distortion. However, PSNR calculation must take the distortion-free original image as the reference standard. In this paper, PSNR is used to characterize the error generated by denoising processing, reflecting the pixel deviation between the original standard image and the denoised reconstructed image. Its calculation formula is as follows:

$$PSNR = 10 \log_{10} \left(\frac{MAX^2}{MSE} \right) = 20 \log_{10} \left(\frac{MAX}{\sqrt{MSE}} \right) \quad (6)$$

The larger the PSNR value, the smaller the distortion and the better the denoising quality. Where MAX is the maximum value of the image pixel color and MSE is the mean square error.

Structural Similarity (SSIM) is mainly used to evaluate the structural similarity between the original image and the denoised reconstructed image. This index uses the image mean to represent brightness information, standard deviation to

represent contrast information, and covariance to measure the degree of image structure correlation. SSIM ranges from -1 to 1. When the two images are completely consistent, the value is 1; the closer the value is to 1, the better the restoration effect and structure preservation effect of the image after denoising. Its mathematical definition is as follows:

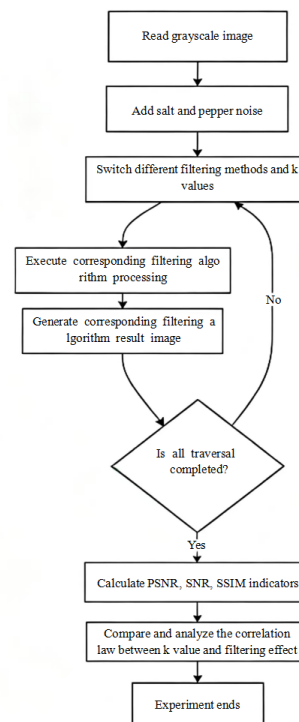
$$SSIM(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x^2 + \sigma_y^2 + c_2)} \quad (7)$$

Where μ_x is the mean of x , μ_y is the mean of y , σ_x^2 is the variance of x , σ_y^2 is the variance of y , σ_{xy} is the covariance of x and y , $c_1 = (k_1L)^2$, $c_2 = (k_2L)^2$

3.3 Experimental Flow

The experimental procedure is as follows: first, the grayscale image is read; second, salt-and-pepper noise is added; third, multiple filtering operations (minimum, order-statistic with $k=2\sim5$, median, maximum, and mean) are applied; finally, the three evaluation metrics are calculated and analyzed.

Fig. 2 Experimental flow chart



IV. Experimental results and analysis

4.1 Comparison of Objective Indicators

Beyond subjective visual inspection, quantitative evaluation metrics are essential for accurate and reproducible assessment of denoising performance, as they avoid individual bias. Salt-and-pepper noise manifests as randomly distributed black and white impulses that degrade texture and edge information. To quantitatively analyze the impact of noise and provide a unified benchmark, we add noise at density 0.05 to the four test images, apply filtering with a 3×3 window, and compute PSNR, SNR, and SSIM relative to the originals.

Fig. 3 Original and 0.05 SP noise-corrupted test images



Table 1: Evaluation metrics of filters on Barbara (0.05 SP noise, 3×3 window)

Filtering Method	PSNR	SNR	SSIM
K=3	25.46	19.14	0.9699
Minimum Filtering (K=1)	12.73	5.23	0.5992
Maximum Filtering (K=9)	11.61	7.86	0.5444
Median Filtering (K=5)	39.40	33.37	0.9986
Mean Filtering	27.39	21.34	0.9775

Table 2: Filter evaluation metrics for Boats (0.05 SP noise, 3×3 window)

Filtering Method	PSNR	SNR	SSIM
K=3	22.71	16.93	0.9331
Minimum Filtering (K=1)	11.72	4.56	0.4910
Maximum Filtering (K=9)	12.15	8.93	0.5301
Median Filtering (K=5)	30.75	25.38	0.9873
Mean Filtering	25.64	20.23	0.9571

Table 3: Filter evaluation metrics for Cameraman (0.05 SP noise, 3×3 window)

Filtering Method	PSNR	SNR	SSIM
K=3	23.98	18.12	0.9686
Minimum Filtering (K=1)	12.18	5.17	0.6360

Maximum Filtering (K=9)	11.42	7.87	0.6019
Median Filtering (K=5)	38.41	32.79	0.9988
Mean Filtering	26.88	21.22	0.9817

Table 4: Filter evaluation metrics for Lena (0.05 SP noise, 3×3 window)

Filtering Method	PSNR	SNR	SSIM
K=3	27.68	22.03	0.9800
Minimum Filtering (K=1)	12.30	6.64	0.5436
Maximum Filtering (K=9)	11.93	6.27	0.5102
Median Filtering (K=5)	35.44	29.78	0.9960
Mean Filtering	26.75	21.09	0.9689

To quantitatively evaluate the salt-and-pepper noise removal effect of different filtering algorithms, this paper selects three indicators of PSNR, SNR and SSIM to compare and analyze the filtering results of four standard test images under the condition of noise density 0.05 and 3×3 window, as shown in Tables 1–4.

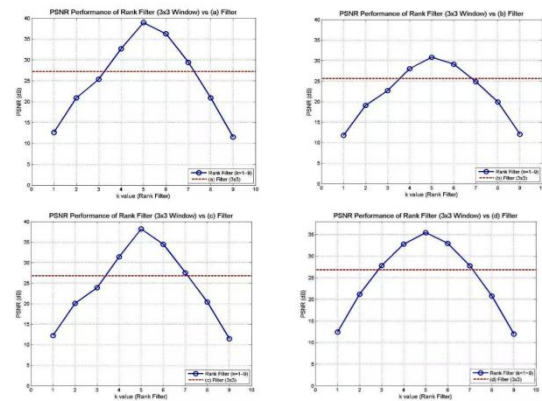
It can be seen from the experimental results that the denoising performance of different filtering methods is significantly different: Median filtering (K=5) has the best performance: the PSNR of the four images is higher than 30 dB, SNR exceeds 25 dB, and SSIM value is close to 1 (up to 0.9988), indicating that it can effectively suppress salt-and-pepper noise while having the strongest ability to preserve image structure and details, achieving the best balance between noise suppression and detail protection. Mean filtering and K=3 order-statistic filtering have moderate performance: the PSNR of mean filtering is generally 25–28 dB, SSIM is 0.95–0.98, with

certain detail blurring; the overall effect of K=3 order-statistic filtering is slightly better than mean filtering, with more prominent detail preservation ability.

Extreme value filtering has the worst performance: the PSNR of minimum filtering (K=1) and maximum filtering (K=9) is lower than 13 dB, SNR is lower than 10 dB, SSIM is generally lower than 0.65, which will cause serious image distortion and is not suitable for salt-and-pepper noise removal.

In summary, under the experimental conditions of noise density 0.05 and 3×3 window, the comprehensive denoising performance of median filtering (K=5) is significantly better than other methods, which is the optimal scheme for dealing with such impulse noise.

Fig. 4 PSNR comparison curves of four datasets under varying k-values: (a) Barbara, (b) Boats, (c) Cameraman, (d) Lena



Under the conditions of noise density 0.05 and 3×3 window, four standard images were tested for denoising with five filtering algorithms. The experimental results show that median filtering (K=5) has the best performance in PSNR, SNR and SSIM, with excellent denoising effect and detail preservation ability; K=3 order-statistic filtering has the second best comprehensive performance, which is better than mean filtering overall; extreme value filtering (K=1, K=9) has the worst indicators and cannot effectively suppress salt-and-pepper noise; mean filtering has general adaptability to salt-and-pepper noise and easily causes image detail blurring. Different image texture complexities lead to small differences in filtering indicators, and median filtering has good robustness in various texture scenes. It can be seen from the PSNR comparison

curve that the evaluation metrics increase initially and then decrease as the k-value rises, and reach the peak at $k=5$, further verifying the optimal denoising effect of median filtering under this parameter for salt-and-pepper noise.

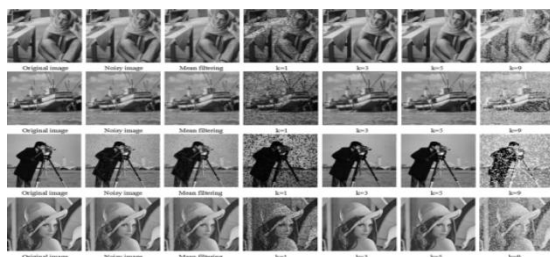
4.2 Visual Effect

The subjective visual results (Fig. 5) are highly consistent with the objective metrics. The original images exhibit clear structural layers and complete textures, whereas the noise-corrupted versions are severely disturbed by diffusely distributed black and white impulses.

The output of minimum filtering ($k=1$) appears globally darker. While white-spot noise is partially suppressed, black-spot noise remains conspicuous, and the overall contrast is severely distorted. As the k-value increases from 2 to 4, image brightness is gradually restored, noise is progressively eliminated, and clarity improves. Median filtering ($k=5$) achieves the most thorough denoising, with almost no visible noise residue, sharp edges, and well-preserved textures. Maximum filtering ($k=9$) yields an overly bright image, with black-spot noise weakened but white-spot noise still prominent and substantial grayscale distortion. Mean filtering reduces noise to some extent but causes severe oversmoothing, resulting in blurred edges, massive texture loss, and a hazy gray appearance.

Overall, across all four test images with varying texture complexities, the subjective visual quality, objective metric tables, and k-value PSNR trends corroborate each other. Median filtering offers the best combination of noise suppression and detail preservation, followed by $k=3$ order-statistic filtering, while extreme-value and mean filtering are insufficient for practical salt-and-pepper noise removal.

Fig. 5 Visual denoising performance of mean filtering and OSF with varying k-values on four benchmark grayscale images



The four rows (top to bottom) correspond to Barbara, Boats, Cameraman, and Lena. For each row, the subimages (left to right) are: original clean image, noisy image (0.05 SP), mean filtering result, and OSF outputs at $k=1$, $k=3$, $k=5$, $k=9$.

It can be seen from the denoising visual effects of four groups of different dataset images in Fig. 5 that the original image has clear layers and complete texture details. After adding 0.05 salt-and-pepper noise, black and white noise points are diffusely distributed in the whole image, and the picture structure is seriously disturbed; combined with the objective indicator data in Tables 1 to 4 and the PSNR comparison line chart of different k-values in Fig. 4, the subjective visual performance is highly consistent with the objective evaluation results.

The two types of extreme value filtering, minimum filtering $k=1$ and maximum filtering $k=9$, show the grayscale distortion characteristics of dark and bright pictures visually respectively, with a large amount of salt-and-pepper noise residue and obvious distortion of edge texture. Corresponding to the data table, their PSNR and SSIM values are at the lowest level, and the PSNR values at $k=1$ and $k=9$ in the line chart are significantly low, further confirming that extreme value filtering cannot effectively suppress salt-and-pepper noise.

With the gradual increase of k-value in order-statistic filtering, the visual perception of the image continues to improve, noise is continuously filtered out, and the picture clarity is gradually improved. $K=3$ order-statistic filtering can better weaken noise and retain basic texture, and the visual effect is better than mean filtering, which is also consistent with the rule that its PSNR and SSIM indicators are higher than mean filtering and extreme value filtering in the table. $K=5$ median filtering has the most thorough denoising, no obvious noise residue, complete and clear image edges, and well-preserved texture details, with the best subjective visual effect. Corresponding to the data table, median filtering has the highest PSNR, SNR and SSIM values on the four images, and the line chart shows that PSNR first increases and then decreases with the increase of k-value, and reaches the peak at $k=5$, fully reflecting the optimal denoising performance of median filtering under this noise density and window condition.

Although mean filtering can dilute salt-and-pepper noise visually to a certain extent, it has serious over-smoothing phenomenon, blurred image edges, massive loss of texture details, and the overall picture is blurred and gray with poor subjective perception, which also matches the characteristics that the indicators of mean filtering are much lower than median filtering and the structural similarity loss is large in the objective data.

Overall, the test images with different texture complexities show the same rule, and the objective indicator data table, k-value PSNR line change trend and subjective visual effect confirm each other. Median filtering has both the best noise suppression ability and detail preservation ability, followed by k=3 order-statistic filtering. Extreme value filtering and mean filtering can hardly meet the denoising requirements of salt-and-pepper noise in both subjective vision and objective quantitative indicators.

V. Conclusion

This study takes four standard grayscale images (Lena, Barbara, Boats, and Cameraman) as experimental subjects, and studies the optimal k-value selection of order-statistic filtering under the unified conditions of 3×3 filtering window and noise density of 0.05, and compares the denoising performance with mean filtering. Order-statistic filtering reconstructs the image based on the gray sorting of neighborhood pixels, with simple algorithm principle, efficient operation and strong engineering practicability. By adjusting the k-value, the algorithm can flexibly switch between minimum, median and maximum filtering, and the imaging effect shows a stable rule with parameters: the image is dark when the k-value is small, and the image brightness increases with the increase of k-value.

Combined with subjective visual effect and objective evaluation indicators of PSNR, SNR and SSIM, it can be concluded that median filtering at k=5 has the best denoising effect, which can effectively remove salt-and-pepper noise while preserving image edges and texture details. In contrast, mean filtering tends to cause over-smoothing of the image, resulting in blurred edges and loss of details. Its performance in noise suppression and detail preservation is weaker than order-statistic filtering under optimal parameters, which cannot meet the requirements of high-precision denoising. The experimental results show

that order-statistic filtering has good adaptability and application advantages for salt-and-pepper noise. This study has certain limitations: only a limited set of images and fixed experimental parameters are used, and the experimental scene is relatively single. Future research can expand the multi-texture standard image dataset, set multi-scale windows and gradient noise density, and explore the adaptive selection strategy of k-value in complex scenes. At the same time, horizontal comparison with a variety of advanced denoising algorithms can be carried out to further optimize the algorithm accuracy and operation efficiency, making it better adapt to engineering scenarios such as medical image processing and industrial image detection.

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